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Nonlinear Active Vibration Absorber Design For Flexible Structures

Lei Chen, Fangpo He, Karl Sammut, and Tan Cao
School of Informatics and Engineering
Flinders University
GPO Box 2100 SA5001, Australia

Abstract- A new technique, called active Vibration Clamping Absorber (VCA), for vibration suppression in flexible structures is proposed and investigated in this paper. The technique uses a Quadratic-Modal-Positive-Position-Feedback (QMPPF) strategy to design a simple second-order nonlinear controller that can suppress the vibration of structures at various resonant points. The proposed QMPPF strategy uses a nonlinear modal control to transfer the vibration energy from the vibrating system to another sacrificial absorber so that large amplitude vibrations in the main structure can be clamped within tolerable limits. The VCA can be constructed using PZT sensors/actuators that are controlled by a DSP controller. The effectiveness of the VCA design based on a QMPPF strategy is validated under multiple-modes control on a flexible vertically-oriented cantilever beam system with a single sensor and actuator. The simulation and experimental results reveal that the proposed strategy is a potentially viable means for real-time control of vibration in large flexible structures.

I Introduction

The application of light materials for the construction of aerospace structures, robots, and automobiles has increased the need to alleviate vibrations in these structures. The challenge for vibration control design is especially significant for some large space structures, such as large solar panel arrays. The difficulty is caused by the structure's inherent characteristics such as: (i) the number of structural vibration modes is infinite in theory and very large in practice; (ii) the structure's natural damping properties are poorly understood and the damping coefficients are very small; (iii) the structure has many low resonant frequencies; etc. Most of the vibration problems in flexible structures are usually related to structural weaknesses that are associated with resonance phenomena, i.e., natural frequencies being excited by external forces. To solve this forms of vibration problems, a variety of control techniques has been proposed, of which modal control is the most widely reported method [1, 2, 3]. The history of modal control as applied to vibration suppression can be generally traced back to 1960s [1]. It became popular in 1970s and 1980s [4] but then fell out of favour in the mid 1980s because of the fundamental shortcomings associated with control spillover and observation spillover [2]. Recently, with the rapid advances in smart structure technology, interest in modal control has again been revived. Inman's research [3] shows that if modal compensation is used as a control law and designed to roll off at higher frequencies, spillover is not a problem. However, most of the control methods used in modal control for flexible structures have focused on linear state feedback or linear output feedback control strategies by using modal position and/or modal velocity. Those methods are very effective for free vibration problems but not for forced vibration problems. It is the objective of the work presented here to extend the modal control method with linear feedback to the modal control with nonlinear feedback by using Quadratic-Modal-Positive-Position-Feedback (QMPPF) to suppress vibrations under primary excitations. In the past decades, the control of free and forced vibrations has largely been focused on classical and modern control methods such as PID or state feedback. From control theory it is known that the solution of differential equations of a system can be separated in two parts, namely, the homogeneous solution and the particular solution.Usually, the particular solution has characteristics that follow the external excitation. In the shape or servo control method [4], the response of a system is always expected to follow the external reference signal as accurately as possible. In vibration control, however, it is always expected that this particular solution is as small as possible and the external excitation can not be treated merely as a disturbance, especially, when a resonant vibration occurs. In some of the design of vibration control problems [3], the reference signal is set to zero as required by the control theory. This condition can only deal with free vibration control or disturbance-induced vibration control. However, a vibration controller that exhibits good free vibration suppression does not necessarily provide adequate forced vibration attenuation. Thus there is a need to find an effective way to suppress forced vibration in flexible structures that are vulnerable to resonant vibrations. On the other hand, attempts to design active vibration control for large flexible structures (LFSs) or distributed parameter systems (DPSs) have been numerous over the last few decades. Due to the necessity of simultaneously controlling many modes, one of the challenges in implementing a proposed vibration controller is to deal with the extensive computations involved. A simple control strategy and a fast controller response are two crucial requirements for real-time implementation of vibration control. In order to meet these requirements, an active Vibration Clamping Absorber (VCA) is designed in this paper. The basic characteristics of the proposed VCA based on QMPPF control include:

- it makes use of the nonlinear modal coupling between the structure and the absorber, to enable the VCA part to absorb the vibration energy from the structure and keep the vibration of the structure within a tolerable range;
- it adds an artificial tunable damper in the controller that targets each mode of interest;
- it is designed and applied only at frequencies where vibrations are most undesirable and it is automatically put into play by the disturbing force in a designed range i.e., outside this range it is inert;
- it employs only modal positions to make the VCA amenable to a strain-based sensing approach;
- it can be implemented in an on-line digital controller.

This paper is organised as follows. The linear dynamic model used here for distributed parameter systems is described in the next section. In Section III, the control spillover and observation spillover is examined in second order Multi-Degree-Of-Freedom (MDOF) modal equations. Furthermore, the Positive-Position-Feedback (PPF) control strategy is represented also in second-order MDOF modal equations. In Section IV, the VCA design is presented and the frequency response and force response of the VCA are demonstrated. In Section V, simulations are carried out for the first, second and third modes. In Section VI, physical experiments are conducted to corroborate the simulation results. The conclusions are presented in Section VII.

II Linear Dynamic Model for DPSs

From the principle of modal control, it is known that the complete dynamic behaviour of a structure can be discretised as a set of individual modes of vibration, each having a characteristic natural frequency, damping factor, and mode shape. By using these modal parameters to represent the system model, problems at specific resonances can be examined and subsequently solved. Here, the class of flexible systems described by the generalised wave equation is considered:

$$\mathbf{m(x)\ddot{w}(x,t)} + 2\zeta\Lambda^{1/2}\mathbf{\dot{w}(x,t)} + \Lambda\mathbf{w(x,t)} = \mathbf{F(x,t)}, \quad (1)$$

which relates the displacement $\mathbf{w(x,t)}$ of the equilibrium position of a body Ω in $L-$dimensional space to the applied force distribution $\mathbf{F(x,t)}$. The operator Λ is a time-invariant symmetric, nonnegative differential operator with a square root $\Lambda^{1/2}$, and its domain $D(\Lambda)$ is dense in the Hilbert space $H = L^2(\Omega)$. The mass density $\mathbf{m(x)}$ is a positive function of the location $\mathbf{x}$ on the body.Without changing the properties of the above system, (1) can be normalised by the change of variables $\longrightarrow$ $\mathbf{w(x,t)/m(x)^{1/2}}$.Here, for simplicity, $\mathbf{m(x)} = I$ in (1) is used. The nonnegative number ζ is the damping coefficient of the flexible system. From the above condition of operator Λ, it is known that its spectrum contains only separated eigenvalues λ_k with corresponding orthogonal eigenfunctions ϕ_k in $D(\Lambda)$, such that $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n, \Lambda\phi_k = \lambda_k\phi_k$, and $\lambda_k^{1/2}\phi_k = \omega_k\phi_k$, where ω_k is the vibration mode frequency and ϕ_k is the corresponding vibration mode shape of the flexible system. According to the nature of Hilbert space, the solutions of (1) can be expressed as:

$$\mathbf{w(x,t)} = \textstyle\sum_{k=1}^{L} v_k(t)\phi_k(x), \quad (2)$$

where, in theory, L should be infinite. However, in practice, it is customary to assume that $w(x,t)$ can be represented with good fidelity by a truncated mode expression of the form of (2) where L may be large but finite. Similarly, the distribution of applied forces can be expanded as:

$$\begin{aligned}\mathbf{F(x,t)} &= \mathbf{F_e(x,t)} + \mathbf{F_c(x,t)} \\ &= \textstyle\sum_{j=1}^{P} f_{ej}(t)\phi_j(x) + \sum_{j=1}^{P} f_{cj}(t)\phi_j(x),\end{aligned} \quad (3)$$

where F_e represents the external excitation forces and F_c represents the control forces provided by point-force actuators. In the following analysis, the case of primary resonances is considered and the external forces can be defined by a set of P ($P < L$) transversal harmonic excitations with amplitude F_j and angular frequency Ω_j close to one of the natural frequencies, i.e. $f_{ej} = F_j\cos(\Omega_j t)$. Therefore, by substituting (2) into (1), the mode amplitudes satisfies:

$$\mathbf{\ddot{v}}(t) + 2\zeta\Delta^{1/2}\mathbf{\dot{v}}(t) + \Delta\mathbf{v(t)} = \mathbf{f}_e(t) + \mathbf{f}_c(t), \quad (4)$$

where $\Delta^{1/2}$ is a $L \times L$ diagonal matrix with diagonal entries $\omega_1, \omega_2, \cdots, \omega_L$, $\mathbf{v(t)} = [v_1(t), \cdots, v_L(t)]^T$, $\mathbf{f}_e(t) = [f_{e1}(t), \cdots, f_{eP}(t)]^T$, and $\mathbf{f}_c(t) = [f_{c1}(t), \cdots, f_{cP}(t)]^T$. By representing the scalar form of (4), any one of the mode amplitude $v_k(t)$ satisfies

$$\ddot{v}_k(t) + 2\zeta_k\omega_k\dot{v}_k(t) + \omega_k^2 v_k(t) = F_j\cos(\Omega_j t) + f_{cj}. \quad (5)$$

III Modal Control and Spillover

Consider the homogeneous (free vibration) problem by setting $\mathbf{f}_e(t)$ equal to zero in (4).Also, consider a state feedback or output feedback control law as indicated by $\mathbf{f}_c(t) = B\mathbf{y}(t)$, where B is the $L \times L$ feedback gain matrix and $\mathbf{y}(t)$ is the measured output of the sensor defined by $\mathbf{y}(t) = C_P\mathbf{v(t)} + C_V\mathbf{\dot{v}(t)}$. Here, C_P is a $L \times L$ matrix indicating the locations of the position sensors and C_V is the $L \times L$ matrix indicating the locations of the velocity sensors. For simplicity, confine the control to position feedback only,thus (4) can be expressed as:

$$\mathbf{\ddot{v}}(t) + 2\zeta\Delta^{1/2}\mathbf{\dot{v}}(t) + \Delta\mathbf{v(t)} = BC_P\mathbf{v(t)}. \quad (6)$$

Since L is quite large in the real applications, it is not possible and also unnecessary to control all L modes.Consequently, N controlled modes are selected with $N < L$ and the amplitude vector $\mathbf{v(t)}$ is partitioned into a controlled $\mathbf{v}_N$ and a residual $\mathbf{v}_R$ part as: $\mathbf{v}(t) = \begin{bmatrix} \mathbf{v}_N \\ \mathbf{v}_R \end{bmatrix}$. Similarly, the system parameters can be partitioned into:

$$\Delta = \begin{bmatrix} \Delta_N & 0 \\ 0 & \Delta_R \end{bmatrix}, 2\zeta\Delta^{1/2} = \begin{bmatrix} 2\zeta\Delta_N^{1/2} & 0 \\ 0 & 2\zeta\Delta_R^{1/2} \end{bmatrix},$$

$$B = \begin{bmatrix} B_N \\ B_R \end{bmatrix}, \text{and } C_P = \begin{bmatrix} C_N & C_R \end{bmatrix}.$$

By using above partitions, (6) can be reorganised as

$$\begin{bmatrix} \ddot{\mathbf{v}}_N \\ \ddot{\mathbf{v}}_R \end{bmatrix} + \begin{bmatrix} 2\zeta\Delta_N^{1/2} & 0 \\ 0 & 2\zeta\Delta_R^{1/2} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}_N \\ \dot{\mathbf{v}}_R \end{bmatrix} + \begin{bmatrix} \Delta_N & 0 \\ 0 & \Delta_R \end{bmatrix} \begin{bmatrix} \mathbf{v}_N \\ \mathbf{v}_R \end{bmatrix} = \begin{bmatrix} B_N C_N & B_N C_R \\ B_R C_N & B_R C_R \end{bmatrix} \begin{bmatrix} \mathbf{v}_N \\ \mathbf{v}_R \end{bmatrix}. \quad (7)$$

Note that while the items on the left side of equations are decoupled, the items on the right side of equations become coupled. The term $B_R C_N$ gives rise to modal coupling due to control action (called control spillover) and the term $B_N C_R$ gives rise to modal coupling due to observation or measurement (called observation spillover). If these terms are large, then the controller's performance and the system's stability will be lost.Although the problem of control spillover and observation spillover has been described in many papers, the problem was stated in state variable form, whereas this explanation is described in second-order MDOF modal equations where the concept is more straightforward and the natural properties of the coefficient matrices are preserved. To the best of the authors' knowledge, this is perhaps the first time that the spillover problem and also the PPF control have been stated in a second-order MDOF form. In order to eliminate the spillover problem in the control system for flexible structures, there are numerous studies that rely on different control algorithms ranging from the simple velocity feedback control law to methods such as the Independent Modal Space Control (IMSC) [5] and the PPF [6]. A comprehensive review of these studies has been included in [4]. The second-order MDOF modal equations developed in the above are again used here to illustrate the advantages of PPF and will be used as a starting point to design the nonlinear modal PPF control in the next section. The basic idea of PPF is to design a modal filter in the second-order MDOF form which rolls off at a high frequency and hence is able to avoid exciting residual modes. This can be explained as follows. The modal filter can be designed as

$$\ddot{\eta}(t) + 2\zeta_f\Delta_f^{1/2}\dot{\eta}(t) + \Delta_f\eta(t) = \Delta_f\mathbf{v}(t), \quad (8)$$

where all the parameters are similar to (4) but are tunable. By feeding back the output η of the modal filter into the structure (6), it leads to

$$\ddot{\mathbf{v}}(t) + 2\zeta\Delta^{1/2}\dot{\mathbf{v}}(t) + \Delta\mathbf{v(t)} = G\Delta\eta(t), \quad (9)$$

where G is a diagonal gain matrix with diagonal entries, such as $g_1, g_2, \cdots, g_L$. Similar to Section III, all the vectors and parameters can be partitioned into N controlled modes and R residual modes, correspondingly. The above two equations (8) and (9) can be rewritten as

$$\begin{bmatrix} \ddot{\eta}_N \\ \ddot{\eta}_R \end{bmatrix} + \begin{bmatrix} 2\zeta_f\Delta_{fN}^{1/2} & 0 \\ 0 & 2\zeta_f\Delta_{fR}^{1/2} \end{bmatrix} \begin{bmatrix} \dot{\eta}_N \\ \dot{\eta}_R \end{bmatrix} + \begin{bmatrix} \Delta_{fN} & 0 \\ 0 & \Delta_{fR} \end{bmatrix} \begin{bmatrix} \eta_N \\ \eta_R \end{bmatrix} = \begin{bmatrix} \Delta_{fN} & 0 \\ 0 & \Delta_{fR} \end{bmatrix} \begin{bmatrix} \mathbf{v}_N \\ \mathbf{v}_R \end{bmatrix}. \quad (10)$$

$$\begin{bmatrix} \ddot{\mathbf{v}}_N \\ \ddot{\mathbf{v}}_R \end{bmatrix} + \begin{bmatrix} 2\zeta\Delta_N^{1/2} & 0 \\ 0 & 2\zeta\Delta_R^{1/2} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}_N \\ \dot{\mathbf{v}}_R \end{bmatrix} + \begin{bmatrix} \Delta_N & 0 \\ 0 & \Delta_R \end{bmatrix} \begin{bmatrix} \mathbf{v}_N \\ \mathbf{v}_R \end{bmatrix} = \begin{bmatrix} G_N\Delta_N & 0 \\ 0 & G_R\Delta_R \end{bmatrix} \begin{bmatrix} \eta_N \\ \eta_R \end{bmatrix}, \quad (11)$$

From (10) and (11), it can be seen that the items on the right side of equations are decoupled. So, there is no control spillover. Moreover, Δ_{fN} can be designed as a low pass filter and Δ_{fR} can be set to zero so that the modal filter rolls off all residual high frequencies. Then, the second equations in (11) and (10) both tend to zero. Combining the first equations in (10) and (11), one obtains

$$\begin{bmatrix} \ddot{\mathbf{v}}_N \\ \ddot{\eta}_N \end{bmatrix} + \begin{bmatrix} 2\zeta\Delta_N^{1/2} & 0 \\ 0 & 2\zeta_f\Delta_{fN}^{1/2} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}_N \\ \dot{\eta}_N \end{bmatrix} + \begin{bmatrix} \Delta_N & -G_N\Delta_N \\ -\Delta_{fN} & \Delta_{fN} \end{bmatrix} \begin{bmatrix} \mathbf{v}_N \\ \eta_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (12)$$

There remains the question of designing the feedback control gain matrix G such that the overall system (12) is stable.

Theorem 1 *The combined system (12) is asymptotically stable if and only if* $0 < g_i < 1, i = 1, \cdots, N$.

Proof

The Laplace transform of (12) leads to

$$(Is^2 + 2\zeta_N\Delta_N^{1/2}s + \Delta_N)\mathbf{V}(s) - G\Delta_N\eta(s) = 0, \quad (13)$$

$$(s^2I + 2\zeta_{fN}\Delta_{fN}^{1/2}s + \Delta_{fN})\eta(s) - \Delta_{fN}\mathbf{V}(s) = 0, \quad (14)$$

where $\mathbf{V}(s)$ and $\eta(s)$ are the Laplace transforms of $\mathbf{v(t)}$ and $\eta(t)$, respectively. Substituting (14) into (13), the characteristic equation of system (12) is

$$det\left|(Is^2 + 2\zeta_N\Delta_N^{1/2}s + \Delta_N)(Is^2 + 2\zeta_{fN}\Delta_{fN}^{1/2}s + \Delta_{fN}) - G\Delta_N\Delta_{fN}\right| = 0. \quad (15)$$

The system is asymptotically stable if and only if all the roots in (15) are in the left side of the s plane. The deployment of (15) can be shown to be

$$Is^4 + \Gamma_1 s^3 + \Gamma_2 s^2 + \Gamma_3 s + \Gamma_4 = 0, \qquad (16)$$

where

$$\begin{cases} \Gamma_1 &= 2\zeta_N \Delta_N^{1/2} + 2\zeta_{fN}\Delta_{fN}^{1/2}, \\ \Gamma_2 &= \Delta_N + \Delta_{fN} + 2\zeta_N \Delta_N^{1/2} 2\zeta_{fN}\Delta_{fN}^{1/2}, \\ \Gamma_3 &= 2\zeta_N \Delta_N^{1/2}\Delta_{fN} + 2\zeta_{fN}\Delta_{fN}^{1/2}\Delta_N, \\ \Gamma_4 &= \Delta_N\Delta_{fN} - G\Delta_N\Delta_{fN}. \end{cases}$$

According to Routh-Hurwitz theorem, system (12) is asymptotically stable if and only if all the principal minors of the Routh-Hurwitz array are greater than zero, i.e.,

$$\Gamma_1 > 0, \begin{vmatrix} \Gamma_1 & 1 \\ \Gamma_3 & \Gamma_2 \end{vmatrix} > 0,$$

$$\begin{vmatrix} \Gamma_1 & 1 & 0 \\ \Gamma_3 & \Gamma_2 & \Gamma_1 \\ 0 & \Gamma_4 & \Gamma_3 \end{vmatrix} > 0, \begin{vmatrix} \Gamma_1 & 1 & 0 & 0 \\ \Gamma_3 & \Gamma_2 & \Gamma_1 & 1 \\ 0 & \Gamma_4 & \Gamma_3 & \Gamma_2 \\ 0 & 0 & 0 & \Gamma_4 \end{vmatrix} > 0.$$

This leads to $0 < g_i < 1, i = 1, \cdots, N$. ■

The above section demonstrates that PPF is very suitable for controlling large numbers of structural vibration modes without causing spillover. However, the theorem developed here has a prerequisite, i.e., system (12) must be homogeneous.When the right side of system (12) has a constant external excitation and the objective is to drive the motion of the structure to zero, it is unlikely that a PPF control is capable to achieve this task.Hence, another approach must be found to effectively suppress the structural vibration under primary external excitations.

IV VCA Design

In this section the active VCA for the structure described in (5) is developed. The purpose of using VCA is to absorb the vibration energy from structure (5) upon which external forces are imposed. To achieve this purpose, a QMPPF control algorithm is designed for both the structure and the VCA. This control force is intended to follow the external force, but with opposite phase. In the later analysis, the effectiveness of the designed QMPPF control algorithm in achieving the above goal will be demonstrated. The design methodology for the active VCA is summarised below. The structure with the external excitation force and the control force can be described by:

$$\ddot{v}_k(t) + 2\zeta_k\omega_k\dot{v}_k(t) + \omega_k^2 v_k(t) = F_j\cos(\Omega_j t) + K_1\omega_k\eta_j^2(t), \qquad (17)$$

and the VCA can be designed as:

$$\ddot{\eta}_j(t) + 2\xi_j\omega_j\dot{v}_j(t) + \omega_j^2\eta_j(t) = K_2\omega_j v_k(t)\eta_j(t), \qquad (18)$$

where η_j represents one of the responses of the VCA, ω_j is the natural angular frequency of the VCA, ξ_j is the damping ratio of the VCA, and K_1 and K_2 are the feedback gains. By using the method of multiple scales [7], one can obtain first-order approximate solutions for (17) and (18):

$$v_k = a\cos(\Omega_k t + \phi_1), \qquad (19)$$

$$\eta_j = b\cos(\frac{1}{2}\Omega_j t + \phi_2), \qquad (20)$$

where a and b are the vibration amplitudes of the structure and absorber, respectively. By defining two detuning parameters τ and σ as $\tau = \omega_k - 2\omega_j and \sigma = \Omega_j - \omega_k$, the modulation equations that govern the amplitudes and phases are given by:

$$\begin{cases} \dot{a} = -\zeta_k\omega_k\xi_j\omega_j a - \frac{K_1}{4}b^2\sin\alpha + f_j\sin\beta, \\ \dot{b} = -\xi_j\omega_j b + \frac{K_2}{4}ab\sin\alpha, \\ a\sigma = -\frac{K_1}{4}b^2\cos\alpha - f_j\cos\beta, \\ b(\tau+\sigma) = -\frac{K_2}{2}ab\cos\alpha. \end{cases} \qquad (21)$$

The parameters α, β, and f_j in (21) are defined as $\alpha = \tau t + \phi_1 - 2\phi_2, \beta = \sigma t - \phi_1$, and $f_j = F_j/4$. The form of the first-order differential equations given in (21) is commonly encountered in the analysis of nonlinear motion of ships and flexible structures [8], where in these studies, the parameters of $\xi_j, \omega_j, \tau, K_1$, and K_2 are fixed and need to be identified from the system.Here this nonlinear phenomenon is capitalised on and employed for the purpose of vibration control. So, those parameters are deliberately designed in the VCA to be tunable for control purposes. When $b \neq 0$, the following can be obtained:

$$a = 4\,\frac{\sqrt{\xi_j^2\omega_j^2 + \frac{1}{4}(\tau+\sigma)^2}}{K_2},$$

$$b = 4(K_1K_2)^{-1/2}\left\{\left[\frac{\tau+\sigma}{2}\sigma - \zeta_k\omega_k\xi_j\omega_j\right] \pm \left[\frac{K_2^2 f_j^2}{4} - (\frac{\tau+\sigma}{2}\zeta_k\omega_k + \xi_j\omega_j\sigma)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}.$$

It is clear that the amplitude of vibration in the structure has nothing to do with the amplitude of external force. The excitation energy has been transfered to the VCA controller.

V Simulation Studies for the VCA

A simple cantilever beam system is selected and used as a research vehicle to evaluate both the above QMPPF control strategy and the VCA controller. In the following studies, only primary resonant excitations are considered.Only one sensor/actuator pair is used for the VCA controller. Although the beam system itself is assumed to be linear and satisfies (5), when it is integrated with the VCA controller it becomes part of a nonlinear system. By using the QMPPF strategy in both the structure and the VCA controller, it will be shown that the energy generated from the external excitation is passed through the structure into the VCA controller. In the first study, the beam system is excited by a sinusoidal signal at a frequency near its first mode of vibration. After the vibration develops into a steady state condition, the VCA controller is switched on. Figure I(a) and (b) show the first mode time-response of the structure and the VCA controller,before and after the controller is turned on while the external excitation is kept constant. The excitation conditions employed with most of the reported studies on vibration control for flexible structures,differ from the study reported in this paper, in that for the former, the excitation is turned off after steady state conditions are attained and the controller is subsequently switched on, while for the latter, the excitation is kept constant. In order to compare the VCA control with PPF control, the first mode time-response of the structure is shown in Figure II, where the same excitation and control gain as in Figure I are applied.

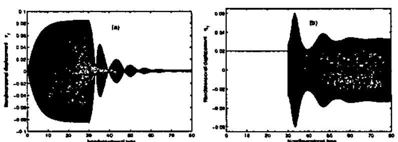

Figure I: Numerical simulations of the first-mode time-response under the VCA control: (a) the sensor response v_1 and (b) the actuator response η_1.

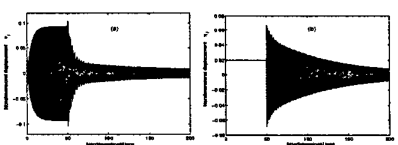

Figure II: Numerical simulations of the first-mode time-response under the PPF control: (a) the sensor response v_1 and (b) the actuator response η_1.

In the second and third studies, the beam system is excited by a

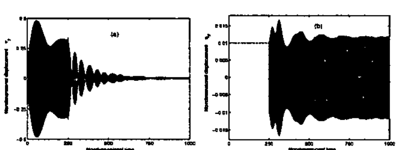

Figure III: Numerical simulations of the second-mode time-response under the VCA control: (a) the sensor response v_2 and (b) the actuator response η_2.

sinusoidal signal at frequencies near its second and third modes of vibration, respectively. Once the vibration develops into a steady state condition, the VCA controller is switched on. Figure III(a) and (b) show the time-responses of the structure and VCA controller at the second mode of vibration. Figure IV(a) and (b) show the time-responses of the structure and VCA controller at the third mode of vibration. From the above simulation studies, it is seen that

- the VCA controller based on QMPPF strategy is very effective for suppression of steady-state primary resonant forced vibrations; and
- the VCA controller has the same performance for each mode regardless of the excitation frequency.

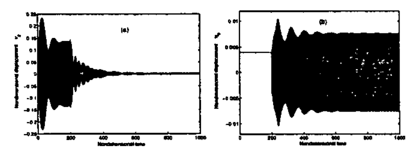

Figure IV: Numerical simulations of the third-mode steady-state time-response under the VCA control: (a) the sensor response v_3 and (b) the actuator response η_3 .

VI Experimental Studies for the VCA

To validate the above control strategy and numerical simulations, a physical test plant was built in the laboratory. The plant is comprised of a 250mm x 13mm x 0.6mm mild steel beam with strain gauge sensors and PZT actuator patches mounted vertically on a 100N shaker, and a dSpace controller. The first three natural frequencies of the beam are experimentally determined to be 11.4Hz, 68.5Hz, and 149.8Hz, respectively. The beam is then subjected to either a single harmonic sinusoidal excitation or a multiple frequency sinusoidal excitation. When the shaker's frequency is tuned to 11.4Hz (i.e., first mode frequency) and the acceleration produced by the shaker is1.2g, the first mode resonance causes large amplitude vibrations. After the vibration is fully developed, the VCA controller is switched on at 20 seconds. Figure V(a) and (b) show the experimental results of the structure's first-mode time-response and the corresponding VCA's time-response. The experimental results confirm the simulation results. and the acceleration produced by the shaker is 1.0g, the second mode resonance causes large amplitude vibrations. Figure VI(a) and (b) show the experimental results of the structure's second-mode time-response and the corresponding VCA's time-response. Again, the experimental confirm the simulation results. It can be seen that the structural vibration has been successfully suppressed even when the external force frequency is at the second resonant mode. When

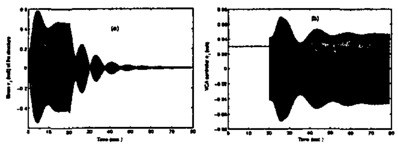

Figure V: Experimental results of the first-mode time-response under the VCA control: (a) the sensor response v_1 and (b) the actuator response η_1.

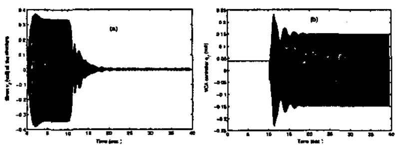

Figure VI: Experimental results of the second-mode steady-state time-response under the VCA control: (a) the sensor response v_2 and (b) the actuator response η_2.

the beam is excited under a multiple frequency sinusoidal excitation with frequencies of 11.4Hz (first mode) and 68.5Hz (second mode), and the acceleration produced by the shaker is 3.0g, the combined resonances cause even larger amplitude vibrations. Figure VII(a) and (b) show the experimental results of the structure's combined-modes time-response and the corresponding VCA's time-response. The experimental results further validate the theoretical analysis. It can be seen that the structural vibration has been successfully suppressed even when the external force frequencies are close to the first and second resonant modes. The Power Spectrum Density analysis is shown in Figure VIII. The suppression effect achieved by the VCA can be more than 30dB attenuation.

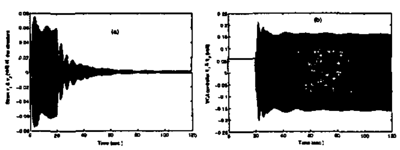

Figure VII: Experimental results of the first- and second-mode time-response under the VCA control: (a) the sensor response $v_1 \& v_2$ and (b) the actuator response $\eta_1 \& \eta_2$.

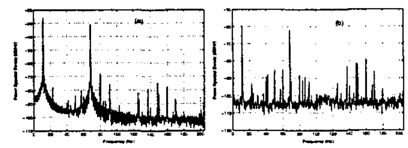

Figure VIII: Power Spectrum Density in the case of first- and second-mode resonant excitations: (a)without the VCA control and (b) with the VCA control.

VII Conclusion

The effectiveness of the VCA design based on QMPPF strategy is validated under multiple modes control on a flexible vertically oriented cantilever beam system with a single sensor and actuator. The simulation and experimental results reveal that the proposed strategy is a potentially viable means for real-time control of vibration in large flexible structures.

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