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**Plausibility of freshwater lenses adjacent to gaining rivers: Validation by laboratory experimentation**

A. D. Werner, A. Kawachi, and T. Laattoe

1School of the Environment, and National Centre for Groundwater Research and Training (NCGRT), Flinders University, South Australia, Australia, 2Faculty of Life and Environmental Sciences, and Alliance for Research on North Africa (ARENA), University of Tsukuba, Ibaraki, Japan

**Abstract** The occurrence of freshwater lenses in saline aquifers adjoining gaining rivers has recently been demonstrated as being theoretically possible by way of analytical solution. However, physical evidence for freshwater lenses near gaining rivers is limited largely to airborne geophysical surveys. This paper presents the first direct observations of freshwater lenses adjacent to gaining rivers, albeit at the laboratory-scale, as validation of their plausibility. The experimental conditions are consistent with the available analytical solution, which is compared with laboratory observations of lens extent and the saltwater flow rate, for various hydraulic gradients. Numerical simulation shows that dispersion can account for the small amount of mismatch between the sharp-interface analytical solution and laboratory measurements. Calibration and uncertainty analysis demonstrate that accurate mathematical predictions require calibration to laboratory measurements of the lens. The results provide unequivocal proof that freshwater lenses can persist despite gaining river conditions concordant with theoretical lenses predicted by the analytical solution, at least within the constraints of the experimental setup.

**1. Introduction**

Freshwater lenses have been encountered in a wide range of terrestrial (noncoastal) groundwater settings, where the underlying saltwater is derived predominantly from evapoconcentration of rainfall [Barrett et al., 2002; Cartwright et al., 2010, 2011; Alaghmand et al., 2013] or paleomarine origins [Kwarteng et al., 2000; Young et al., 2004; Milewski et al., 2014; Houben et al., 2014]. Freshwater lenses in riverine settings are usually associated with losing river conditions, which may occur temporarily under episodic river events [Bauer et al., 2006; Cendon et al., 2010] or persist under normal river conditions thereby potentially creating vast freshwater lenses [Cartwright et al., 2010, 2011]. Freshwater lenses found in aquifers adjacent to rivers that are gaining under normal flow situations are typically linked to transient causal factors, such as bank storage effects, floodplain overtopping, or anthropogenic impacts, and are usually presumed to be dependent on the frequency of inflow events and permanency of freshwater sources [Bates et al., 2000; McCallum et al., 2010; Alaghmand et al., 2015].

The occurrence of stable freshwater lenses in saline aquifers adjoining gaining rivers, as shown schematically in Figure 1, appears unlikely under normal flow conditions in the river, and in the absence of other freshwater sources. That is, buoyancy forces (i.e., due to freshwater-saltwater density differences) and the groundwater hydraulic gradient appear prima facie to act toward the river, and would thereby be expected to flush freshwater from the aquifer into the river. However, Werner and Laattoe [2016] recently developed an analytical solution based on principles commonly applied to coastal aquifers to show that freshwater lenses can theoretically occur in aquifers adjoining gaining rivers (see Figure 1) due to the effects of buoyancy. They compared the horizontal dimension of freshwater lens extents obtained from their analytical solution to airborne electromagnetic (AEM) survey results [e.g., Viezzoli et al., 2009] for the River Murray floodplains (Australia). The results provided some evidence that the two methods are in broad agreement, although validation of the analytical solution was impeded by uncertainty in the AEM data and a lack of piezometer-based measurements of freshwater lenses in the River Murray system (and elsewhere).

Given that evidence for stable freshwater lenses adjacent to gaining rivers is to date based almost exclusively on a mathematical premise, validation of Werner and Laattoe’s [2016] analytical solution is warranted. In
particular, their solution is based on simplifying assumptions (e.g. sharp-interface between freshwater and saltwater, a stagnant freshwater lens, etc.) that have not been tested, and the limited field evidence for terrestrial freshwater lenses is not adequate to confirm the physicality of their solution. To address this, the present study aims to reproduce the stable freshwater lenses posed by Werner and Laattoe [2016] through controlled laboratory experiments using a physical model of saline groundwater flow toward a freshwater river. Physical models exploring variable-density flow phenomena are relatively common [Jakovovic et al., 2011; Chang and Clement, 2012, 2013; Morgan et al., 2013], and include studies of the freshwater lenses of oceanic islands [Stoeckl and Houben, 2012; Dose et al., 2014]. The intent is to present the first depictions of buoyant freshwater lenses in a laboratory-scale reproduction of an aquifer adjoining a gaining river, thereby complementing previous physical modeling of other mixed convection problems.

In the present study, the Werner and Laattoe [2016] solution is tested through inverse modeling and uncertainty analysis, which are designed to explore the range of freshwater lens extents that might be predicted without direct measurements of the lens. This approach aims to provide insight into the uncertainty of direct application (i.e., forward modeling) of the Werner and Laattoe [2016] theory, which is likely to be applied without calibration to calculate freshwater lens characteristics in real-world settings, and thereby will rely on field-based estimates of hydrogeologic properties due to the limited field observations of freshwater lenses. Studying this type of recently discovered flow system, i.e., a stable freshwater lens over flowing saltwater, is important not only for understanding riparian and floodplain salt transport processes, but there are likely to be coastal settings in which analogous conditions exist.

2. Methodology

2.1. Experimental Apparatus and Materials

The sand tank used to conduct freshwater lens physical experiments is shown in Figure 2. An unconfined aquifer is bounded by two fluid reservoirs (freshwater on the left and saltwater on the right), which connect to the sand through screens of fine copper mesh housed in plastic frames. The screen housing is 11 mm
thick. The screens themselves are approximately 1 mm thick and almost flush with the side of the housing in contact with the sand. The tank has plate glass on the front face and plastic at the back, sides, and bottom. The tank’s internal dimensions are 697 mm long, 350 mm high, and 31 mm wide. The reservoirs have a horizontal dimension of 35 mm, leaving approximately 605 mm in length for the sand. Two ports allow for fluid supply and drainage into and out of the reservoirs.

Fluid was supplied from constant-head tanks (20 L plastic Mariotte bottles) placed at fixed elevations and connected to the reservoir ports using 10 mm internal diameter silastic tubing. One Mariotte bottle containing saltwater was connected to both ports of the right-hand reservoir, while four interconnected Mariotte bottles [see Werner et al., 2009 for details of air and fluid connection arrangements] supplied freshwater inflow to the lower port of the left-hand reservoir. The upper port of the left-hand reservoir drained mixed water at a controlled elevation, thereby establishing the head and maintaining the low salinity and density of the freshwater reservoir. Here, we refer to this reservoir as “freshwater” despite that it contains a mixture of saltwater and freshwater, although the mixing is dominated by freshwater flushing. A similar approach was used with success in sand tank experiments of seawater intrusion by Goswami and Clement [2007]. Recirculation in the left-hand reservoir was maintained by a head difference of a few centimeters between the freshwater Mariotte bottles and the discharge tube.

Saltwater was produced by dissolving 1.543 kg of calcium chloride dihydrate (CaCl₂·2H₂O) into 18.834 kg of triple distilled water at 25°C in a 20 L plastic bottle. Based on textbook values [Weast et al., 1985] for the CaCl₂ solution concentration of 58.275 mg L⁻¹, the density was expected to be 1046 kg m⁻³. A portable density meter (Densito 30PX, Mettler-Toledo International Inc.) provided a density of 1046.2 ± 0.1 kg m⁻³. The electrical conductivity (EC) of the saltwater solution was 88.3 ± 0.1 mS cm⁻¹ measured with an EC probe (TPS WP-84 v1.0 Conductivity-Salinity-Temp Meter⁴). The solution was colored with a red food dye (Pillar Box Red Food Colour, 50 mL, Queen Fine Foods Pty Ltd) using 10 mL of dye per 1 L of salt solution to facilitate visual monitoring of saltwater behavior. Experimental photography was shot using a tripod mounted Nikon® D3100 14 megapixel digital SLR camera. The effect of the dye on density was undetectable by the density meter and changes to EC were negligible, that is, within the error margins of the meter. Tracer adsorption is unlikely to have impacted the results because steady state conditions were sought [e.g., Jakovovic et al., 2012]. The density and EC of the freshwater used in the experiments were 999.9 ± 0.1 kg m⁻³ and 0.582 ± 0.005 mS cm⁻¹.

The porous material used in the experiments was the same medium sand (30 CFS grade, supplied by Sloan Sands, Dry Creek, South Australia) that was used by Mannicke [2010] and Jakovovic et al. [2012]. They applied three different approaches to determine the sand’s range of freshwater hydraulic conductivity (K): grain-size analysis, Darcy column testing, and in situ sand tank testing, and both dense and loose packing arrangements were evaluated. These produced K values of 54–138, 25–88, and 50–55 m/d, respectively. Jakovovic et al. [2012] adopted a final K value of 49.25 m/d in their modeling of the physical experiments of saltwater upconing. We undertook further in situ testing of the sand prior to freshwater lens experiments, as discussed in section 2.2.

The geometric variables of the experiment and apparatus are illustrated in Figure 3. The origin of the x and z dimensions is denoted by O; x₀ is the sand length; xᵢ is the extent of freshwater lens; ηₑ and ηᵢ are saltwater and freshwater lens thicknesses, respectively; ηₑᵣ and ηᵢᵣ are, respectively, saltwater and freshwater lens thicknesses adjacent to the freshwater reservoir; ηᵢᵣ and ηᵢ are the depths of the saltwater and freshwater reservoirs, respectively. Not shown in Figure 3, W/L is the width of the tank perpendicular to flow. Figure 3 also illustrates the screen cross-section, whereby the screen occupies approximately 22% of the sand cross-sectional area. The screen is considered to behave in a surrogate manner to resistive riverbed material, which is included in the equations provided by Werner and Laattoe [2016] (as discussed below).

2.2. Experimental Procedure

To obtain a relatively uniform porous medium, a wet-packing method was adopted [Goswami and Clement, 2007; Ojuri and Ola, 2010]. In situ testing of the sand was undertaken prior to the freshwater lens experiments by imposing several different rates of steady freshwater flow (Qᵢ) through the sand tank. The depths of freshwater in the inlet and outlet reservoirs (ηᵢᵣ and ηᵢᵣ, respectively) were used within the equation for unconfined steady state flow:
K' is the value of the sand tank's bulk hydraulic conductivity, which accounts for any losses through screens. Values of K' ranging from 38 to 45 m d\(^{-1}\) were obtained, taking into account repeated testing and the uncertainty of measurements.

The sand K can be derived from in situ testing by treating the tank as a series of three materials (inlet screen, sand, outlet screen), assuming the screen hydraulic conductivity equals that of the sand, and treating the reduction in area through screens as a proportional reduction in W (i.e., screen W = kW, where k is ratio of the screen area to the sand tank's cross-sectional area), as:

\[
K = \frac{2Qf}{W(\eta_f^2 - \eta_t^2)} \left( 1 + \frac{2x_{\text{screen}}}{\kappa x_0} \right) \quad (2)
\]

Taking the screen thickness (x_{\text{screen}}) as 1 mm and k as 0.22, and comparing equations (1) and (2), the conversion between K and K' is: \(K = 1.015K'\). This conversion equation does not account for any increased resistance to flow created by the screen hydraulic conductivity. The screen resistance has the effect of further reducing \(\kappa\), which is considered in the calibration and uncertainty analysis phases.

In freshwater lens experiments, the sand tank was initially filled with flowing freshwater, and subsequently, saltwater was introduced under constant head conditions until the lens reached a steady state condition, as determined by comparing the photographed lens extent at 30 min intervals. The head of freshwater inflow was modified to ensure that the reservoir representing the river boundary had a stable water level and remained adequately fresh.

Two sets of experiments (A and B) were performed for repeatability, each comprising three different head drops and saltwater flow rates between the reservoirs, as listed in Table 1. Head differences were created by maintaining the right boundary (inlet) head at 0.310 m, and modifying the left boundary (outlet) head. The time-varying parameters measured at regular intervals during experiments included \(x_L\), \(\eta_f\), and the EC of the water exiting the freshwater reservoir \(EC_f\). Application of a simple mixing equation allowed for an estimate of the saltwater discharge \(Q_s\) as:

\[
Q_s = Q_o \frac{EC_o - EC_f}{EC_s - EC_f} \quad (3)
\]

where \(EC_o\) and \(Q_o\) are the EC and discharge of the left-hand reservoir (i.e., representing the "river"), and \(EC_s\) is the EC of saltwater. Table 1 lists \(\eta_f\), \(EC_o\), \(Q_o\), and \(Q_s\) for the six experiments, where each value is the mean obtained from multiple readings.
2.3. Application of Freshwater Lens Theory

The analytical solution for a freshwater lens adjacent to a gaining river proposed by Werner and Laattoe [2016] is based on a stagnant freshwater lens overlying flowing saltwater. This differs from analytical solutions applied to coastal settings where the saline groundwater is presumed stagnant and the freshwater discharges to the sea [e.g., Bear et al., 1999]. Otherwise, the assumptions of Werner and Laattoe [2016] are similar to those adopted in studying coastal settings [e.g., Werner and Simmons, 2009], including a sharp freshwater-saltwater interface, steady state conditions, homogeneous aquifer, etc. Werner and Laattoe [2016] provide an equation for the saltwater discharge, as:

$$q_s = \frac{K}{2} \left( \frac{\rho_f}{\rho_s} \eta_s^2 - \eta_b^2 \right)$$

(4)

Here, $q_s$ is the discharge per unit width of aquifer (in units L$^2$ T$^{-1}$), and $B_s$ and $K_r$ are the thickness and hydraulic conductivity of the riverbed (or outlet screen in the laboratory experiments), $\rho_s$ is the saltwater density, and $\rho_f$ is the freshwater in both the lens and the river (or the freshwater reservoir in the laboratory experiments).

The depth of saltwater adjacent to the river boundary is given by Werner and Laattoe [2016]:

$$\eta_{br} = \sqrt{-\frac{2B_s q_s}{K_r \left(1 - \frac{\rho_f}{\rho_s}\right)}}$$

(5)

The lens extent can be obtained by combining equation (4) with Werner and Laattoe’s [2016] equation (7), to produce:

$$x_L = \left( \frac{\rho_f}{\rho_s} \right) x_0 - \frac{KB_s}{K_r} \left( \frac{\eta_b^2}{\eta_s^2} - 1 \right)$$

(6)

The freshwater lens thickness at the river boundary is obtained from equation (5) and using the relationship $\eta_{fr} = \eta_s - \eta_{br}$.

In seeking to relate the sand tank setup to field-scale parameters, dimensionless parameters are sought from equations (4) to (6). For example, a nondimensional lens length $x_L'$ is proposed on the basis of equation (6), as:

$$x_L' = \frac{x_L}{x_0} \left( \frac{\eta_b^2}{\eta_s^2} - \frac{\rho_f}{\rho_s} \right)$$

(7)

In equation (5), a mixed-convection ratio is apparent that corresponds with Abarca et al.’s [2007] “dimensionless freshwater flux,” except in terms of saltwater rather than freshwater, as:

$$a = \frac{q_s'}{K\delta}$$

(8)

Where $q_s'$ is specific saltwater discharge [in units L T$^{-1}$], given as $q_s/\eta_s$, $\delta$ defines the buoyance force, given as $1 - \rho_f/\rho_s$. A dimensionless representation of the interaction between the streambed (outlet screen in the laboratory experiments) and the aquifer is drawn also from equation (6), as:

$$b = \frac{KB_s \eta_b^2}{K_r x_0 \eta_s^2}$$

(9)
be consistent across experiments were assigned unique values, whereas parameters that could reasonably differ between experiments were allowed to vary. On the basis of Table 1 results, it is clear that the hydraulic properties of the tank changed between the two set (A and B) of experiments (i.e., lower \( q_s \) for a given head difference in the B set of experiments). We attribute this to the effects of compaction, which Männicke [2010] showed could have a major influence on \( K \) for the same medium sand. Also, some level of compaction is apparent in the form of slight lowering of the sand surface between Experiment sets A and B.

The calibration process was moderated through regularization [Alcolea et al., 2006; Knowling et al., 2015], in that any deviation from best estimates of laboratory parameters arose only where it improved the calibration match to experimental observations: \( x_L \), \( \eta_f \), and \( q_s \). Thus, the calibration objective function was the weighted sum of model-measurement deviations (squared) from both laboratory parameters and experiment observations. Calibration was performed with the Evolutionary Solving Method (ESM) in Microsoft Excel® because it outperformed other solvers in terms of reducing the calibration objective function. This is likely due to the highly nonlinear character of the underlying equation, as illustrated by Werner and Laatatto [2016], and the attributes of the ESM approach [Ragsdale, 2015].

The parameters modified through model calibration, and the allowable range of parameter variation (based on measurement uncertainty and repeatability) are given in Table 2. \( K \) was estimated from equation (2) and was therefore dependent on calibrated values of \( K' \) and \( \kappa \). \( \kappa \) (screen resistance) in equations (4)-(8) was assigned a value of \( \kappa K \), thereby accounting (in surrogacy) for both cross-sectional reduction and screen resistance.

The uncertainty analysis aimed to find plausible ranges in the experimental lens extent that take into account laboratory parameter uncertainty, that is, in the absence of measurements of the lens extent. This was achieved using a similar approach to the calibration process, except the objective functions for estimating minimum and maximum \( x_L \) comprised the sum (across all experiments) of \( x_L \) or \( 1/x_L \), respectively. In seeking the \( x_L \) minimum and maximum, the optimization process was regularized by adding to the objective function the sum of the weighted deviations (squared) between the parameters used to obtain minimum and maximum \( x_L \), and those from the calibration. The parameter limits adopted in the uncertainty analysis were the same as those used in the calibration process, as listed in Table 2.

### Table 2. Calibration Parameters, Best-Estimate Values, and Parameter Constraints

<table>
<thead>
<tr>
<th>Laboratory Parameter</th>
<th>Experiment</th>
<th>Best Estimate</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K' ) (m d(^{-1}))</td>
<td>All</td>
<td>4.15</td>
<td>±0.5</td>
</tr>
<tr>
<td>( x )</td>
<td>All</td>
<td>0.22</td>
<td>0.013–0.32</td>
</tr>
<tr>
<td>( \eta_f ) (m)</td>
<td>All</td>
<td>0.605</td>
<td>±0.001</td>
</tr>
<tr>
<td>( B_j ) (m)</td>
<td>All</td>
<td>0.001</td>
<td>10(^{-6})–0.002</td>
</tr>
<tr>
<td>( W ) (m)</td>
<td>All</td>
<td>0.031</td>
<td>±0.001</td>
</tr>
<tr>
<td>( \eta_f ) (m)</td>
<td>All</td>
<td>0.310</td>
<td>±0.001</td>
</tr>
<tr>
<td>( q_s ) (m d(^{-1}))</td>
<td>A-1, A-2, A-3</td>
<td>0.300, 0.290, 0.280</td>
<td>±0.001</td>
</tr>
<tr>
<td>( B_1, B_2, B_3 )</td>
<td>0.305, 0.295, 0.285</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_s ) (kg m(^{-3}))</td>
<td>All</td>
<td>1046.2</td>
<td>±0.1</td>
</tr>
<tr>
<td>( \rho_f ) (kg m(^{-3}))</td>
<td>A-1, A-2, A-3</td>
<td>1001.3, 1002.1, 1003.0</td>
<td>±0.1, ±0.1, ±0.5</td>
</tr>
<tr>
<td>B-1, B-2, B-3</td>
<td>1000.9, 1001.3, 1001.6</td>
<td>±0.2, ±0.2, ±0.1</td>
<td></td>
</tr>
</tbody>
</table>

#### 2.4. Calibration and Uncertainty Analysis Approach

Experimental parameters were calibrated to obtain an optimal match between measurements and analytical model values of lens extent (\( x_L \) and \( \eta_f \)) and saltwater discharge (\( q_s \)) for the six experiments (Table 1). The intent of the calibration exercise was to ascertain whether processes and/or experimental error unduly modified the observed lens extent relative to the analytical solution result. Parameters expected to

#### 2.5. Numerical Model Evaluation of Dispersion Effects

An extension to the sharp-interface analysis is undertaken to briefly explore dispersive effects using the SEAWAT numerical code [Langevin et al., 2008], which has been widely validated for similar applications. SEAWAT’s mathematical formulation is contained in the software documentation and is omitted for brevity. Experiment A-1 is used as the basis for exploring dispersion effects on the lens extent and accompanying physical processes. The model grid was designed to maximize the number of cells while maintaining a reasonable computational burden, thereby achieving a balance between accuracy and model run times. The grid represented the sand tank dimensions, using cell sizes of approximately 2.5 mm long by 2.5 mm deep by 8 mm wide, resulting in a grid of 120,528 cells. The saltwater reservoir was represented by specified-head and concentration boundary conditions, while the freshwater reservoir was simulated using the General-Head Boundary package (GHB) of SEAWAT [Langevin et al., 2008]. This allowed for an identical
replication of the analytical solution conductance approach (i.e., $K_f/B_r$) to the simulation of the sand tank's downstream mesh. Salinities adopted within GHB representations of the freshwater reservoir in the various experiments were taken from the experimental measurements listed in Table 1, thereby accounting for the mixture of freshwater and saltwater contained therein. Dispersive simulations adopted longitudinal and transverse dispersivities of 0.1 and 0.01 m, respectively, based on values used in other experiments of similar scales [e.g., Jakovovic et al., 2011; Badaruddin et al., 2015]. Parameters were otherwise as listed above.

3. Results

The steady-state lenses from the laboratory experiments are shown in Figure 4, which also illustrates the analytical solution results arising from calibration and uncertainty analysis. From visual inspection, the calibrated analytical solution generally reproduces the experimental lens extents, in particular for Experiments A-2, A-3, and B-1, which show very close matches. There is slight underestimation of the lenses of Experiments A-1, B-2, and B-3. In Experiments B-2 and B-3, the uncertainty analysis range is unable to capture the observed lens extent, thereby indicating that exogenous factors may play an important role in the experimental results. Unfortunately, head measurements in sand tank piezometers were not sufficiently accurate to use in a comparison to the analytical solution, due mainly to capillary rise and the relatively small size of the tank. The quantitative assessment of the experimental observations, as given below, provides a more systematic review of the results.

Table 3 compares measured experimental parameters, parameter combinations, and lens characteristics with those obtained from the calibration and uncertainty analysis. The results show that the lens length and
Table 3. Measured and Calibrated Parameter Values, and Uncertainty in Lens Extents

| Experiment | \(x_L\) (m) | \(\eta_L\) (m) | \(|q_s|\) (m d\(^{-1}\)) | \(K\) (m d\(^{-1}\)) | \(\kappa\) | \(K/B\) (10\(^3\) d\(^{-1}\)) | \(\delta\) (10\(^{-2}\)) | \(\eta_L/\eta_f\) | \(x_f\) (m) | \(W\) (m) |
|------------|-------------|----------------|----------------|----------------------|---------|----------------|----------------|--------------|------------|--------|
| A-1        | 0.200       | 0.155          | 0.447          | 41.5                 | 0.220   | 9.27            | 4.29           | 1.03         | 0.605      | 0.031  |
| Cal.       | 0.193       | 0.129          | 0.424          | 48.0                 | 0.021   | 0.68            | 4.31           | 1.03         | 0.606      | 0.032  |
| Unc.       | 0.137, 0.269| 0.100, 0.297   |               |                      |         |                |                |              |            |        |
| A-2        | 0.060       | 0.075          | 0.711          | 41.5                 | 0.220   | 9.27            | 4.22           | 1.07         | 0.605      | 0.031  |
| Cal.       | 0.066       | 0.068          | 0.694          | 48.0                 | 0.021   | 0.68            | 4.21           | 1.07         | 0.606      | 0.032  |
| Unc.       | 0.041, 0.150| 0.043, 0.286   |               |                      |         |                |                |              |            |        |
| A-3        | 0.015       | 0.020          | 0.975          | 41.5                 | 0.220   | 9.27            | 4.13           | 1.11         | 0.605      | 0.031  |
| Cal.       | 0.016       | 0.022          | 0.937          | 48.0                 | 0.021   | 0.68            | 4.19           | 1.11         | 0.606      | 0.032  |
| Unc.       | 0.000, 0.094| 0.000, 0.273   |               |                      |         |                |                |              |            |        |
| B-1        | 0.415       | 0.210          | 0.266          | 41.5                 | 0.220   | 9.27            | 4.33           | 1.02         | 0.605      | 0.031  |
| Cal.       | 0.403       | 0.236          | 0.215          | 41.9                 | 0.089   | 2.02            | 4.36           | 1.01         | 0.606      | 0.032  |
| Unc.       | 0.280, 0.417| 0.220, 0.304   |               |                      |         |                |                |              |            |        |
| B-2        | 0.227       | 0.170          | 0.428          | 41.5                 | 0.220   | 9.27            | 4.29           | 1.05         | 0.605      | 0.031  |
| Cal.       | 0.182       | 0.198          | 0.417          | 41.9                 | 0.089   | 2.02            | 4.32           | 1.04         | 0.606      | 0.032  |
| Unc.       | 0.144, 0.197| 0.185, 0.294   |               |                      |         |                |                |              |            |        |
| B-3        | 0.142       | 0.135          | 0.536          | 41.5                 | 0.220   | 9.27            | 4.26           | 1.09         | 0.605      | 0.031  |
| Cal.       | 0.106       | 0.167          | 0.615          | 41.9                 | 0.089   | 2.02            | 4.28           | 1.08         | 0.606      | 0.032  |
| Unc.       | 0.092, 0.123| 0.158, 0.283   |               |                      |         |                |                |              |            |        |

\(a\) Measured lens characteristics and best-estimate parameter values given in rows labeled "Mea."

\(b\) Calibrated values given in rows labeled "Cal."

\(c\) Maximum and minimum \(x_L\) and \(\eta_L\) obtained from the uncertainty analysis given in rows labeled "Unc."

thickness of the experiments are reasonably well matched by the analytical solution, consistent with the lens depictions in Figure 4. In particular, the mismatch in \(x_L\) is less than 10% in Experiments A-1, A-3, and B-1, and the mismatch in \(\eta_L\) is less than 10% in Experiments A-2 and A-3. Measured values of \(x_L\) and \(\eta_L\) differ to calibrated results by up to 25.6% and 23.4% (i.e., in Experiment B-3), respectively. The calibration mismatch in \(|q_s|\) is 19.1% and 14.7% in Experiments B-1 and B-3, respectively, but otherwise, \(|q_s|\) mismatch is <5% and shows no relationship to \(x_L\) and \(\eta_L\) mismatches. Some biases are evident in the calibration results, including the underestimation of \(|q_s|\) in Experiment set A, and the underestimation of \(x_L\) and the overestimation of \(\eta_L\) in Experiment set B.

The inability of calibration to obtain small mismatch in some of the \(x_L\), \(\eta_L\), and \(q_s\) observations led to parameters that coincide with the limits of measurement error, which are given in Table 2. For example, \(K\) and \(W\) are at upper limits in all cases of Experiment set A, \(\eta_L\) is at its lower limit in A-2 and A-3, and \(\delta\) is at the upper limit in A-1 and A-3. These results are consistent with the underestimation in \(|q_s|\) from the calibration of Experiment set A, in that higher \(K\), \(W\), and \(\delta\), and lower \(\eta_L\) will create stronger saltwater discharge, notwithstanding the complex interplay of other parameters in calibrating the analytical solution to \(x_L\) and \(\eta_L\). In Experiment set B, \(W\) and \(\delta\) are at their upper limits and \(\eta_L\) is at its upper limit in B-2 and B-3.

The uncertainty analysis results provide important experimental insights. For example, measured values of \(x_L\) and \(\eta_L\) are within the ranges obtained from the uncertainty analysis for Experiments A-1, A-2, and A-3, and therefore, measurement error can reasonably account for the calibration mismatch of \(x_L\) and \(\eta_L\) in Experiment set A. However, \(x_L\) and \(\eta_L\) fall outside of the uncertainty analysis ranges for Experiments B-1 (\(\eta_L\)), B-2 (\(x_L\) and \(\eta_L\)), and B3 (\(x_L\) and \(\eta_L\)). Specifically, in Experiment set B, \(x_L\) and \(\eta_L\) are, respectively, underestimated and overestimated by the analytical solution, despite optimization attempts to seek plausible upper and lower limits to \(x_L\) and \(\eta_L\) (i.e., without violating parameter constraints imposed on the basis of measurement error estimates). The ranges in \(x_L\) and \(\eta_L\) obtained from the uncertainty analysis, in combination with some parameter values at their limits, support the indication from visual inspection that exogenous factors may have played an important role in the experiments. This is explored to some degree using numerical modeling of Experiment A-1, and examined in greater detail in the Discussion.

The results from the numerical modeling of Experiment A-1 is provided in Figure 5, which illustrates the salinity distributions and flow velocities of both dispersive and nondispersive representations of the laboratory conditions.

The nondispersive numerical model interface location is an improvement over the analytical solution, particularly adjacent to the freshwater reservoir, where the laboratory and numerical results produce almost identical interface locations. There is freshwater recirculation within the freshwater lens in both dispersive...
and nondispersive cases, as identifiable from the freshwater flowing from left to right within the lens, and subsequently flowing parallel to the mixing zone toward the freshwater boundary. This is analogous to seawater recirculation in coastal aquifers [Smith, 2004]. Recirculation requires dispersion [Abarca et al., 2007], and therefore, recirculation in nondispersive models is a function of the artificial numerical dispersion that is unavoidable in applying SEAWAT with all dispersion parameters set to zero [Langevin et al., 2008]. Dispersive freshwater velocities were around six times the rates of nondispersive freshwater velocities.

Figure 5 shows that the mixing zone near the water table in the dispersive case has shifted to the left relative to the nondispersive case. This reflects the nature of the mismatch in Experiment set B, and from this it can be concluded that dispersion was the primary cause of mismatch in those experiments. There is also a strong vertical component of velocity in the saltwater region near the downstream boundary that is the most likely cause of analytical solution error in the mixing zone location near the freshwater boundary, given that the sharp-interface analytical solution neglects vertical flows.

4. Discussion

4.1. Experimental Sensitivities

Experimental sensitivities, in addition to those given by Werner and Laattoe [2016] for freshwater lenses more generally, are determined using the analytical solution to show important links between parameters and experimental results. Formulations for $\eta_{fr}$ and $x_L$, on the basis of equations (1), (2), (4), and (6), can be developed, as:

$$x_L = x_b - \left( x_b + \frac{B_t}{K} \right) \left( \frac{\eta_{fr}^2 - 1}{\eta_{fr}^2 - \rho_s/\rho_f} \right) \cdot \eta_{fr} = \eta_{fr} - \frac{\eta_{fr}}{\sqrt{\eta_{fr}^2 + 1}}$$

From these, $x_L$ and $\eta_{fr}$ are shown to be independent of $K$ and $W$, in particular where $K$ is used to represent the hydraulic conductivities of the experiments. Furthermore, the stream bed resistance term $B_t/K$ is added to the density ratio ($\rho_f/\rho_s$) and boundary head ratio ($\eta_{fr}/\eta_b$) as correlations that are readily identifiable in earlier equations. Sensitivity analysis adopts a 0.1% change in the parameters of Experiment B-1, and logarithmic sensitivities [Kabala, 2001] are used to normalize parameter and output changes, as listed in the table below.

Table 4 results highlight the role of density and head differences in controlling lens characteristics, relative to other parameters, as evident in their higher sensitivities. Parameter correlations are demonstrated by
logarithmic sensitivities that are almost the same in magnitude but opposite in sign. Parameter correlations and insensitivities that are evident in the underlying equations (as discussed above), are apparent in Table 4 (e.g., $x_L$ and $\eta_x$ are insensitive to $K^r$).

The negative sensitivity of $|q_s|$ to $\kappa$ appears somewhat counterintuitive, given that the screen resistance $K_s$ is proportional to $\kappa$, and therefore increasing $\kappa$ would be expected to increase $|q_s|$ (i.e., a positive sensitivity). However, the calibration process involved modifying $K'$ and $\kappa$, and therefore an increase in $\kappa$ produces a lower $K'$ and a higher $K_s$ (for a given $K'$), on the basis of equation (2). Hence, the lowering of $|q_s|$ with increasing $\kappa$ (i.e., a negative sensitivity) is a nuance of the calibration methodology.

4.2. Investigation of Error

To further investigate sources of error, including the potential influence of exogenous factors, an additional calibration was undertaken in which parameter limits were systematically removed. As expected, the unconstrained calibration reduced the mismatch between measured and calculated values of $x_L$, $\eta_x$, and $|q_s|$. That is, the average absolute errors in $x_L$, $\eta_x$, and $|q_s|$ obtained from the constrained calibration are 0.018 m, 0.020 m, and 0.032 m$^2$ d$^{-1}$, respectively, compared to unconstrained-calibration errors of 0.001 m, 0.004 m, and 0.004 m$^2$ d$^{-1}$, respectively. This error reduction is accompanied by significant changes to parameters. In Experiment set A, $\kappa$, $K'$, $K/B_s$, and $\delta$ changed by 105%, 15%, 47%, and $-6\%$ to $-29\%$, respectively, while in Experiment set B, the same parameters changed by $-31\%$, $-22\%$, $-62\%$, and $62\%$ to $69\%$, respectively. These changes led to $K'$ and $\delta$ exceeding their measurement error limits. Specifically, the calibrated value of $K'$ for Experiment set A was 55 m d$^{-1}$, exceeding its measurement error upper limit of 48 m d$^{-1}$, and $\delta$ ranged from 0.030 to 0.073 across all experiments, whereas the measurement error limits of $\delta$ were 0.041 to 0.044.

The increase in $K'$ to 55 m d$^{-1}$ is relatively subtle given the naturally high variability of hydraulic conductivity, and is arguably within acceptable experimental bounds, particularly given the challenges of creating homogeneous sand in a laboratory sand tank. Conversely, the changes to $\delta$ that occurred within the unconstrained calibration are difficult to justify from a physical standpoint. The water densities used in the experiments were rechecked, and revealed no further information to assist in understanding the calibrated range of $0.030-0.073$. The most likely explanation is that $\delta$ values obtained from unconstrained calibration are providing a surrogate role for other errors, e.g., in other parameters or in the underlying assumptions of the analytical solution. On the basis of the formula for $x_L$ in equation (10), $\rho_f/\rho_s$ may potentially compensate for errors in $\eta_s^2/\eta_x^2$ in seeking the measured value of $x_L$ through model calibration, although both $\eta_b$ and $\eta_x$ were easily measured to a high level of accuracy, and did not change significantly in the unconstrained calibration.

The unconstrained calibration results indicate that unquantified factors may have influenced the experimental results, including processes and experimental nuances that are not considered elsewhere in this paper. For example, despite the apparent sharpness of the saltwater-freshwater interface in the photographs of laboratory experiments, the effects of dispersion appear from numerical modeling to have nonetheless influenced lens extents (e.g., see Figure 4). Pool and Carrera [2011] found that density effects could account for discrepancies between dispersive and sharp-interface representations of the seawater extent in a coastal aquifer. They modified the density term using a correction that contained the transverse dispersivity to improve the calculation of the penetration of the saltwater front. Hence, the absence of dispersivity in the analytical solution may similarly present as an error in the calibration of the water density contrast, as has occurred here. Attempts to apply the correction of Pool and Carrera [2011] to the current results are beyond the scope of this manuscript, and is the subject of a concurrent body of research.

Dispersion, in combination with buoyancy, controls seawater circulation in coastal aquifers [e.g., Post et al., 2013; Qu et al., 2014]. Similar mechanisms cause entrainment of freshwater in discharging saltwater within the experimental freshwater lens, as shown by velocity vectors in Figure 4 that represent circulating freshwater. As with the effects of dispersion, the circulation of freshwater within the lens is not considered in the analytical solution, which presumes a stagnant lens.

Other potential sources of experimental error include the nonuniform geometry of the screen housing, head losses through the mesh of the inflow reservoir, aquifer heterogeneity, and unsaturated zone effects. However, given the close match of the numerical model with laboratory observations, these are likely to
Morgan et al. [2013] report significant unsaturated zone effects on their transient sand-tank experiments of seawater intrusion overshoot, whereby larger capillary zones served to significantly increase the saturated thickness of their otherwise shallow aquifer. In the current modeling, the water table is rather close to the sand surface, as shown in Figure 4, and therefore, unsaturated zone effects are expected to be small. Notwithstanding this, Figure 4 shows that the interface clearly continues above the water table into the capillary fringe, and on this basis, there is clearly some effect of the unsaturated zone on the lens. Indeed, our sand tank observations are the first to report a freshwater-saltwater interface that is clearly within the capillary fringe, although the analysis of this requires more sophisticated methods than the analytical solution that is tested here.

An evaluation of the effect of the screen that connects the inlet reservoir to the sand was undertaken by manipulating equation (6) such that the upstream screen was treated as an adjoining unconfined aquifer of length $B_r$ and hydraulic conductivity $K_r$. That is, the same parameters as adopted for the downstream screen. The resulting equation is given below:

$$x_L = \left( x_0 - B_r + \frac{B_r}{4} \right) \left( \frac{1}{\sqrt{\pi}} - \frac{1}{\sqrt{\pi}} \right) \left( \frac{\left( 1 - \frac{1}{\sqrt{\pi}} \right)}{\sqrt{\pi}} \right)$$

(11)

Calibration of the observed experimental lens using equation (11) and corresponding formulae for $\eta_0$ and $|q_s|$ produced very similar results to the calibration described earlier, and hence we retain the original formulae of Werner and Laattoe [2016] for the purposes of demonstrating the application of unmodified forms of their equations.

## 4.3. Laboratory Versus Field-Scale Conditions

The dimensionless variables $x_0$, $a$ and $b$ allow for comparison between field conditions and the laboratory setup. Table 5 presents dimensionless parameter values for both the laboratory setup and various combinations of field-based parameters for the River Murray, as reported by Werner and Laattoe [2016].

The comparison of dimensionless numbers in Table 5 shows that laboratory freshwater lenses have a similar dimensionless lateral extent relative to the River Murray conditions reported by Werner and Laattoe [2016]. However, compared to field conditions, the laboratory experiments involved significantly higher saltwater fluxes relative to the buoyancy force, on the basis of $a$. A wide range of streambed-aquifer conditions ($b$) are apparent in the data of Werner and Laattoe [2016], whereas the laboratory experiments have $b$ values in the lower range of field parameters, implying that the laboratory streambed resistance (i.e., the outlet screen) was relatively weak in the face of strong saltwater flows.

## 5. Conclusions

Sand tank experiments have been undertaken to provide a physical basis for the theoretical premise of buoyant freshwater lenses next to gaining rivers proposed by Werner and Laattoe [2016]. Their analytical solution guided the experimental design conducted under controlled conditions in a laboratory sand tank. Six experiments are reported that apply different head gradients between the fresh and saltwater reservoirs, each producing a freshwater lens at steady state. The photographic images of each lens provide the first documented evidence that a buoyant freshwater lens may persist adjacent to a gaining river in a laboratory setting.

The analytical solution, subjected to regularized calibration, provided a reasonable match to the laboratory observations of freshwater lens extent, using parameters that remained within the limits of measurement error. A small amount of bias in the analytical solution estimates of freshwater lenses occurred, whereby
three of the experiments showed underestimated lens extents and overestimated lens thicknesses. Numerical simulation shows that this aspect of the experimental-analytical solution mismatch is attributable to the effects of dispersion.

An uncertainty analysis was performed to ascertain minimum and maximum lens extents for each experiment that might plausibly be obtained within the bounds of parameter error, while ignoring the direct lens measurements. The wide range of lenses that are obtained within experimental error limits indicates that direct measurements of lenses are essential to calibrate the analytical solution, and that application of the method on the basis of parameter estimates is likely to lead to uncertain predictions of freshwater lenses. The uncertainty range also showed that measurement error could account for the mismatch observed in the first set of three experiments but not the second, in which dispersion is the most likely cause. It is suggested that factors exogenous to the analytical solution play an important role in the experiments, and/or some of the assumptions of the analytical solution have been violated. Consequently, an unconstrained calibration was performed in which parameter limits were systematically removed to investigate further sources of error. The unconstrained calibration results provided significantly reduced mismatch between measured and calculated lens characteristics at the expense of unjustifiable changes to parameters, specifically the density contrast. In accordance with the findings of Pool and Carrera [2011], calibration tended to modify density to account for mismatch, i.e., between the sharp-interface analytical solution and the experiments, that was attributable to the effects of dispersion.

An extension of Werner and Laattoe’s [2016] sensitivity analysis was performed to investigate links between parameters and experimental results. The results demonstrate the dominating influence of density and head differences in controlling lens characteristics. In addition, dimensionless variables were developed to facilitate comparison between the experimental setup and field conditions. Lateral extents of the lenses observed along the River Murray reported by Werner and Laattoe [2016] matched those of the experiments. However, the flux of saline water to the river relative to the buoyancy force and the screen resistance were found to be at the upper and lower ends (respectively) of parameters encountered in field settings. This analysis further strengthens the case for the occurrence of freshwater lenses adjacent to gaining reaches on the saline floodplains of the River Murray and provides valuable insight for prospecting similar lenses adjacent to other rivers.

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