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On the Finitude of Time in Aristotle and Leibniz

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This paper considers an argument called here the Shift argument, which has the upshot, it will be argued, that time cannot be infinite into the past but must have had a beginning or first instant. The Shift argument can be found in Aristotle and many others. We largely consider versions due to Aristotle and Leibniz, with the aim to draw essentially opposite conclusions from Aristotle.

1. Introduction

We need a good metaphysical argument for the finitude of time. By the finitude of time, I do not (just) mean the temporal finitude of the universe of matter, but of time itself. I take this to be the thesis that there is no time earlier than a certain finite duration of time ago, nor any time later than a certain finite duration into the future.

Philoponus has such an argument: the traversal argument. As George Couvalis (2013) presents it, it aims to show that both the universe and time are finite. I find the prospect of such a conclusion exciting. However, I must also confess that I find the traversal argument unconvincing. So, I want to try on a different argument which I will call The Shift Argument (the name comes from Nerlich, 2013). This argument has a long history, it’s in Aristotle, Augustine, Proclus, Philoponus, Maimonides, Leibniz, Kant, Russell and others, and discussed extensively by Nerlich (e.g. Nerlich, 2013). Interestingly, as we will see, it is put to differing uses. (For a careful survey of the many ancient sources see Davidson, 1987).

The methodology of this paper is less historical than philosophical, to engage with Aristotle and others as philosophers, with the intent to reach interesting philosophical conclusions about time.
2. Aristotle’s argument for the temporal infinitude of the universe

Aristotle argues as follows (for a good account, see Seeskin, 2005: Ch. 3).

**Premise One:** time is infinite.

**Premise Two:** if time is infinite then the material universe is infinite in time.

**Conclusion:** the universe is infinite in time.

We consider this in following sections. We will end up employing a version of Premise two to argue for the opposite conclusion.

**Premise one:** Time is infinite. Immediately, three qualifications need to be made. Strictly, a more rigorous statement of Premise one is that: time is potentially infinite in respect to addition. First, as is well known, Aristotle did not believe in actual or completed infinities. He thought that a thing can be infinite only in the sense that it can never be completed, one can always go one step further. This is termed potential infinity. Second, Aristotle saw even further than this, in that he realised that there is a difference between two sorts of (potential) infinity: infinity by addition (something can always be added); and infinity by division (parts can always be further divided, this is what modern logicians call dense). An example of the first in the natural numbers 0,1,2,...; while an example of the second is the real numbers between 0 and 1. According to Aristotle, time is infinite in both senses, whereas in contrast space is infinite by division but not by addition. Third, one further qualification is needed. There is a kind of surrogate infinity by addition to be obtained if one keeps dividing a dense thing, say dividing in halves, in that this process can go on forever. But Aristotle did not mean this about infinity by addition. The idea is that things of the same size can always be added. This property characterises the natural numbers but not the reals between 0 and 1. In sum: time is potentially infinite in respect to addition. (For a useful discussion see Ross, 1923:85.)

Aristotle’s argument for the infinitude of time is different according to different commentators, and disappointingly weak. Ross (1923:84) describes it as: if time were finite there would be a beginning and an end to time, which is an “impossible result” (*Physics*, 206a10). Seeskin offers that the idea of a first instant of time is “incoherent” (2005:26). In another place, Seeskin says of Aristotle’s view: time is the number of motion, but time is inconceivable apart from the now (2005:67). The now unites the beginning of the future with the end of the past, hence there must be moments on either side of the now. Seeskin criticises this as well-known to be arbitrary, since there is no obvious reason that there couldn’t have been a first moment. Copleston (1946:66) says that for Aristotle time is (potentially) infinite since it can be added to indefinitely, but time never exists as an actual infinite. Unfortunately, this is more-or-less just a statement of infinity by addition. So Aristotle is not very convincing as to why we should believe Premise one.
3. Aristotle’s second premiss: The shift argument

**Premise Two:** If time is infinite then the material universe is infinite in time (The consequent means that at an additively infinite number of times, the material universe exists at those times).

**Proof:** The shift argument: if it could be that time were infinite but matter were only finite in time, then the universe could have begun to exist at another time, such as three minutes earlier.

> Why, after an infinity of not being, was it generated at one moment rather than another? (Aristotle *On The Heavens*, 283a 11–14)

It is wrong to suppose that the universe could have existed at times other than it did: to postulate that the material universe might exist at different times in an otherwise empty time, is an absurdity.

Aristotle’s conclusion is that the universe is infinite in time. We will develop this argument in the following sections.

4. Philoponus and Maimonides

As a preliminary, we note that Philoponus and especially Maimonides dispute Aristotle’s conclusion. In contrast to Aristotle, they are finitists about time and the universe. Unlike Aristotle, they give the second premiss a theological reading. Seeskin (2005: 79) reports that Maimonides, following Proclus, raises Aristotle’s problem of how, if time could be infinite and the universe finite, God could have made the universe at a time other than when it actually started. Aristotle would like to ride the Shift argument to infinitude. But Maimonides argues that, unlike we humans, God has no impediments on action. Thus, no reason need be given for why God’s free action should be now and not earlier or later.

> Now as the Creator ... has no incentives necessitating the alteration of a will, nor hindrances or impediments that supervene or cease to exist, there is no reason in respect of which He should act at one time and not act at another; but on the contrary His action exists, just as his permanence does, permanently in actu. (*The Guide of the Perplexed* II, 14)

Maimonides’ method is characteristic of him, and a technique of metaphysicians everywhere: he argues for the possibility of his desired metaphysical conclusions, rather than their truth. He hopes to show that the Shift argument does not go through for a God acting freely and without impediments (“self-motivated” [Seeskin, 2005:83]). Hence, hopefully, Aristotle’s argument cannot have probative force, it does not establish its conclusion. Maimonides is formally correct here, but for one matter. The flaw lies in the theological rendering itself. Aristotle, as we here interpret him, is contending that there is no difference between a temporally infinite universe beginning at one rather than another. Maimonides argues, under the additional
assumption of God, that there can be a difference. This does not show that Aristotle’s argument as he gave it is unsatisfactory, especially given that Aristotle himself does not import a theological premiss in support. In short, Aristotle’s argument is still standing after this criticism.

We see later that Leibniz likewise imports a theological version of the Shift, but it turns out not to be crucial for his argument.

5. An Aristotelian back-up for the shift argument

Aristotle has several backup arguments to support the Shift, particularly arguments from proportionality and from possibility. Though I do not rely on these for my own argument, it is worthwhile to notice his argument from proportionality or ratio, since it is quite modern in its orientation.

In making this point, Aristotle states his main shift insight in another way.

But that which is produced or directed by nature can never be anything disorderly: for nature is everywhere the cause of order. Moreover, there is no ratio in the relation of the infinite to the infinite, whereas order always means ratio. But if we say that there is first a state of rest for an infinite time, and then motion is started at some moment, and that the fact that it is this rather than a previous moment is of no importance, and involves no order, then we can no longer say that it is nature’s work: for if anything is of a certain character naturally, it either is so invariably and not sometimes this and sometimes of another character (e.g. fire, which travels upwards naturally, does not sometimes do so and sometimes not) or there is a ratio in the variation. (Aristotle Physics, 252a 10–20)

A modern interpretation is that Aristotle is arguing that nature has orderly laws, and laws of nature are expressed in equational form, typically something like \( f(x) = k \cdot g(x) \), where \( x \) is a suitable parameter, such as distance or time, and \( k \) is a constant. But equations lend themselves to expressions as ratios: this equation becomes \( \frac{f}{g} = k \). But, and here is the very modern point, if we attempt to describe laws of nature applying to the infinite, such as the infinities of time obtaining before material nature begins, we are required to describe infinities as yielding ratios, and there is still to this day no good theory of how such infinities obey arithmetical calculations such as division, thereby yielding ratios or proportions.

The Shift argument then appears as the conclusion that anyone postulating infinities such as infinite periods of time has a disorderly universe to deal with: there is no way to stipulate a difference between the universe beginning at one time or another.

6. Leibniz and the shift argument

We turn now to consider a different application of the Shift, due to Leibniz. It will be seen that it winds its way around Aristotle’s argument in interesting ways. Our conclusion will be that it can be used to support an argument for the finitude of time.
Leibniz famously uses the shift, but he aims to use it for a different purpose: against spatial and temporal realism, especially in the Fourth Letter to Clarke (Alexander, 1956:36–44, esp. paras 15–16). Like Maimonides, he gives it a theological presentation. He says “it is a like fiction ... that God might have created the world some millions of years sooner”. The issue at stake here is between spatial or temporal realism, as against relationism. Realism here is the thesis that space and time are composed of (existing) atomic parts, the points of space or the instants of time. Relationism is the thesis that neither points nor instants nor intervals nor other purely spatial nor temporal items exist in themselves, there are only “certain orders of things”, that is spatial relationships between objects and temporal relations between events. Thus, Leibniz is arguing for relationism and against realism.

If realism is true, then points of space and instants of time are separate existences from the things/events that occupy them. In which case the “at” relation can be varied: a material event could have been at a time 3 min earlier than it is. Russell’s interpretation of it is that if realism about space is true, the world could be rotated relative to space. Intuitively, this results in no difference at all. Yet the realist must say there is a difference. Leibniz’s conclusion is thus: realism, and its consequence that the “at” relation could be different, is therefore false.

Realism/relationism is a very different agenda from infinitude/finitude. Why doesn’t Leibniz deduce the finitude of time from this? Can a relationist believe in finite time? I will answer these questions by proposing a positive account of the finitude of time.

7. A positive account

Suppose, for proof of contradiction, that Aristotle is right and time is infinite. Now we have to ask what is the relationship between the universe and time. There are two possibilities: either (1) the universe is finite (in time), or (2) the universe is infinite. Now, if (1) the universe is finite, then we can immediately apply Aristotle’s own Shift argument, to conclude a finite universe within infinite time can be shifted in thought to a different instant in time. But it surely affronts our sense of reality to think that these states of affairs are different!

This is the key observation in the shift argument. Leibniz makes it out by appealing to somewhat different principles: The principle of the identity of indiscernibles has it that things which are not discernibly different, are identical. The principle of sufficient reason has it that things do not happen without sufficient reason. While not specifically appealing to God, it also has a theological version which does: God does not create a difference without a reason. Given any of these, the identity of the universe at different times follows. Of course, these principles can and have been challenged, (Nerlich, 2013). But there is a deeper intuition independent of them, I think, namely that any alleged differences of times in these circumstance are bogus. I must confess that my intuitions scream out that this is so. Bare differences in times, with no other consequences, are no differences.
We turn now to the second horn of the dilemma above. Suppose (2), that the universe is infinite. Let us say that the universe “fills up time” if matter and space exist at every time. Now of course if the universe does not fill up time, then it seems clear that there ought to be spare moments to shift the universe to. But in fact, it really does not seem necessary for this argument that the universe not fill up time. If time is infinite then (as a definition of infinity) time has a 1–1 correspondence with a proper subset of itself, so that the universe could be shifted to a proper subset of its original times. Say, for example, let the infinite set 0,1,2,... be shifted to coincide with the set 3,4,5,... by skipping the first three instants. Alternatively, if the universe is temporally ordered like the integers: ... -2,-1,0,1,2,..., then shift the universe three temporal units in one or other direction.

In passing, it should be noted that we are appealing to modern physics and logic here to develop the point about infinity. It goes without saying that the author takes their account as well-entrenched and correct. But Aristotle would presumably not like this talk about apparently completed infinities. Whether his view is suitable for development as modern mathematics remains to be seen. (Thanks to a referee for emphasising this point.)

The conclusion seems inescapable. Irrespective of whether the universe fills up time, if time is infinite, a shift argument goes through, and we are landed with the absurdity that there is a difference where intuition says there are none. Hence, time is finite; and as a bonus the universe is finite in time (it could hardly be any other way!).

8. A complication

So much the worse for infinite time. We have achieved this result by applying a shift argument to Aristotle's postulation of infinity for time. In so doing, we have turned Aristotle around to achieve the opposite conclusion from Aristotle's core argument. We can also observe that it is just as well that we could see that Aristotle's positive arguments for infinite time were weak, or we might have a paradox in the wings. But now we remind ourselves that there is another very similar shift argument in the offing, namely the anti-realist argument for relationism as advanced by Leibniz and Russell. Must we buy this desirable conclusion of finite time only at the cost of losing realism?

Leibniz and Russell, we note, were more overtly concerned with realism about space than realism about time. And this certainly has been Nerlich's main concern too. It is no surprise, then, that, in the name of realism, Nerlich finds fault with the shift argument applied to space. His reply to the shift argument is that in a variably curved space or space-time, varying the “at” relation in general does not leave the new state of affairs indiscernible. Following general relativity, space-time is generally variably curved. The variable curvature means that space-time is not uniformly curved (and uniformity was an explicit assumption of Leibniz, fourth letter para. 18). Failure of uniform curvature ensures that as things are moved around, they change
their shape, which ensures the failure of indistinguishability. For example, the sums of the angles of triangles vary, and this can be measured. Thus, the principle of the identity of indiscernibles is by-passed rather than disputed because there is nothing indistinguishable to be identified.

We might object that flat space or flat space-time wouldn’t yield this result, and we note that Leibniz includes as an explicit premiss that space is uniform or homogeneous (in our terms, space is of constant curvature). Nerlich replies that the most general assumption is that space and spacetime are curved. Flat space is an exception. To argue as Leibniz does is to be committed to saying that uniform spaces “alone among possible spaces are senseless monstrosities” (emphasis in the original, Nerlich, 2013:24).

This is tricky. Nerlich is aiming to show that Leibniz’s argument for relationism does not go through, rather than to refute relationism outright. To avoid Nerlich’s counterexample, Leibniz has to add the extra premise of uniformity, which he believed to be true, but which is definitely false in the real world, though doubtless true in some non-actual world. That is presumably enough for our purposes to rebut Leibniz.

In addition, Nerlich (1976) has a positive argument against relationism and in favour of realism: the handedness argument, also to be found in Kant. Consider the difference between an otherwise identical pair of hands, a left hand and a right hand. They cannot be rotated or translated to coincide, they are handed. This phenomenon is widespread: the whole of stereochemistry depends on the chemical differences that follow from the difference between left-handed molecules and right-handed molecules. Now, strictly speaking, it is the space in which these things are embedded that allows or forbids the property of handedness. Space is said to be orientable if handedness is permitted, otherwise space is non-orientable. To see how space might be non-orientable, imagine left and right hands flattened out in 2D but embedded in a Mobius strip. Then one of the hands can be translated all the way around the strip so that it does coincide with the other. A Mobius strip is thus non-orientable. Contrast with the hands embedded in a dog-collar. There is no way to translate one the hands while staying in the surface, so that it coincides with the other. A dog-collar is thus orientable.

What this shows is that the acknowledged differences between handed things depend not on the relations between the handed things, but on the topological properties of the embedding space as a whole: IF there are handed things, THEN space is orientable. Affirm the extra premise that there are handed things, and we have the realist conclusion. There seems to be no way that a relationist can avoid this conclusion. Orientability is a property of spaces as a whole, not of the relations between things in it. At the same time, the extra premise that there are handed things is required, and it is not entirely obvious that it is true. A pair of hands close-by to one another is not immediately sufficient to establish that there are handed things, since it might be possible to take one of them on a long journey as with the Mobius strip, a journey that brings it back to coincide with the other of the pair. Thus a further strengthening of the premise would be indicated. We leave this for another day.
To gather our thoughts: if Leibniz's argument fails, then it does not immediately follow that realism is true, nor that the universe is infinite. The realist position needs a positive argument such as the handedness argument. If the premise that there are handed pairs is sustained as it seems to, then realism is sustained.

If Leibniz-style of argument holds, then we have a proof of the finitude of time and the universe. But it does not hold. However, the failure of the argument is enough to establish a required conclusion in the debate about time, namely realism about space-time and matter.

Given that basis, we can re-raise the question of the finitude of time. Is there a way to avoid the counterargument to the Shift argument?

We can illustrate the common strategy by appeal to Einstein's field equation for general relativity.

\[ R_{\mu\nu} - \frac{(g_{\mu\nu}/R)}{2} + g_{\mu\nu}\Lambda = \left(\frac{8\pi G}{c^4}\right)T_{\mu\nu} \]

This looks formidable but isn't really. It relates geometry on the left hand side (curvature \(R\) and metric \(g\), which are geometrical properties of spacetime), directly to the distribution of matter on the right hand side (stress-energy or mass-energy tensor, that is \(T\)).

In particular, there are the vacuum field solutions where matter = 0. One example is empty Minkowski space-time, there are others. However, note that for the typical case, where geometry is locally variable, translation of the distribution of matter across a varying metric-and-curvature will mean that the matter ceases to fit the space-time. This manifests itself as change in shape of the matter, produced by tidal stresses. Hence, Nerlich's objection continues to hold: there will be no indiscernible shifts. The \textit{a priori} argument for finite time remains blocked.

There is, however, a quick way, not so \textit{a priori}.

We can, in thought, divide time from “the universe”, which we can understand as space-matter. Space-matter is not such an obscure term; space and mass-energy fall nicely under the material universe as we understand it. A piece of space-matter is an (everywhere locally spacelike) time-slice of 4D space-time. It even has a mass and volume. Just as what was leftover after space-time was mass-energy, so here what is leftover is just time.

Consider now that Nerlich's argument places too stringent demands on what is a Shift. Nerlich insists that the shift consists in shifting matter across space-time. But of course, if curvature is not uniform this is impossible: space-time structure carries, and is carried by, matter. They are locked together. But surely this cannot be a fair rendering of the Aristotle/Leibniz demand to explain why it didn't start sooner. A Shift would be impossible just because the matter would have to embed itself in flat, vacuum-field space-time: leave the space-time fixed and shift the mass-energy and you will not have a fit that satisfies the equations of GR. Yet the spirit of Aristotle is surely to ask why the space-matter universe isn't earlier. That's different. For that, we take note of the featurelessness of infinite time (with or without geometry/space...
but definitely without mass) stretching infinitely far ago. In short, it isn’t quite the identity of indiscernibles. It is rather that there is a featureless region in space-time that should be collapsed by intuition to no more than one featureless slice. Again, think of a stack of space-like slices of space-matter. Think of there being an infinite sandwich of vacuum slices to precede the first matter-event. Why didn’t space-matter start earlier?

So I think that Aristotle’s intuition is sound here, and Nerlich’s objection rather misses the point if applied to infinite time. But what we can conclude is considerable: infinite time and finite space-matter are incompatible. Together they invite us to contemplate the possibility of a situation containing multitudes which are not really different from one another. Either both are infinite or both are finite. But which?

I threatened to depart from the a priori, and here I go. The answer is rather apparent: the universe of matter is to all appearances finite. That’s the simple message that our best current physics gives us. That is the premise that the argument needs. It follows that time is likewise finite into the past, which is just what we want. This gives out that there is something distinctly odd about the idea of a finite universe preceded by infinite time. That surely appeals to the intuition.

**Conclusion**

Is this conclusion rather disappointing in that it does not exploit the full force of the a priori? Not really. The Aristotelian shifts argument is a necessary part of the argument. And would we not want it? It essentially says that an infinite time with a temporally finite universe is a nonsense. For then the choice is cut down. The shifts argument enables us to put down the possibility of an infinite past, in favour of a less fanciful idea.
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