Lobbying, Campaign Contributions, and Electoral Competition

Thanh Le*  Erkan Yalcin†
Flinders University  Flinders University

Abstract
This paper studies the effect of lobby groups on electoral competition and equilibrium policy outcomes employing a ‘money for policy favours’ model of lobbying. Our results show that when a lobby group seeks to influence an electoral outcome, it will make a financial contribution to only one political party whose policy is closely aligned to its own ideal policy. When misappropriation of campaign funds occurs, political parties that divert more funds for personal gain stand on more independent platforms and require larger contributions from lobby groups. Greater electoral competition could reduce policy distortions but this, in turn, sparks more intense lobbying thereby increasing the scope of misappropriation of funds. In the case of multiple lobbying, political parties either demand different levels of campaign contributions or leave them with different levels of satisfaction.

JEL Classification: D72; D82
Keywords: Special Interest Politics; Lobbying; Political Competition; Campaign Contributions

1 Introduction
Lobbying has long been an integral part of political decision-making in representative democracies. By making political contributions, special interest groups gain important favours from or access to legislators over a preferred policy. In other words, money contributions serve as an incentive device

*College of Business, Government & Law, Flinders University, Bedford Park, SA 5042, Australia. Email: thanh.le@flinders.edu.au. Corresponding author.
†College of Business, Government & Law, Flinders University, Bedford Park, SA 5042, Australia. Email: erkan.yalcin@flinders.edu.au.

This paper studies the effect of lobby groups on electoral competition and equilibrium policy outcomes by presenting a model of lobbying in which special interest groups donate campaign funds to political candidates running for election who then embezzle some of those contributions. Specifically, we consider the environment of two polar parties competing in an electorate that consists of informed and uninformed voters. To attract votes from an uninformed electorate, the financially constrained parties engage in a costly advertising and campaigning program. At first, we examine the case of a single special interest group seeking to influence the policy position of political parties by offering contributions that are contingent upon the policy proposed by each party. We assume that self-interested politicians derive utility from income and holding office. The parties, therefore, have an incentive to propose policies which attract financial contributions. With regard to the campaign funds received, the candidates are induced to choose policy platforms closer to those of the donating interest groups than what they would otherwise pursue. Upon accepting these campaign funds, the political candidates allocate an amount to election campaigning in order to enhance their likelihood of winning and another amount to their personal pursuits. Since there is no legally binding contract between the political party and the lobby group, the donor is unable to prevent politicians from diverting funds to non-campaign uses.

In deciding on the proportion of political contributions to be allocated to various uses, the politicians confront a trade-off. On the one hand, increasing the funds used for campaigning raises the probability of winning the election. On the other hand, money diverted for personal purposes yields utility (i.e. rents), irrespective of the election outcome. The proportion of funds diverted for personal use is considered as one form of misappropriation, and is defined as the degree of embezzlement. In the electoral competition game, the lobby group announces its contributions to the parties upon observing the party characteristics (i.e. ideological preferences, etc.). Once the parties choose their policy platforms, the contributions are paid. After that, the election is

---

1The fact that constrained politicians rely on support from lobby groups to relax their budget constraint has been pointed out by Hall and Deardorff (2006). In particular, they view lobbying as legislative subsidies of different forms ranging from legislative drafting, research reports and testimonies to campaign contributions and campaigning. In this paper, we do not model the process in which lobbying involves information gathering or policy implementation. Instead, we focus on strategic transfers of financial resources from special interest groups to policy-makers.
We obtained several interesting results in this setting. In particular, we found that the interest group will never lobby more than one party. This is either because of high financial costs or high ideological barriers that prevent it from approaching the other party. Once the interest group and its aligned party decide to negotiate with each other on the policy to be implemented and the contribution level, an agreement between them will be reached. Such an agreement is unique. In accord with this agreement, the equilibrium policy is responsive to a change in either the political party’s popularity, the lobby group’s ideal policy or the fraction of uninformed voters. It is also affected by a change in the degree of embezzlement. Here, the embezzlement of political contributions is likely to have two conflicting effects on lobby group donations. On the one hand, lobbying is less productive when funds intended for campaigning are diverted for personal use, so that the lobby group may have less incentive to make political contributions. On the other hand, there is a sequential game effect that makes the lobby group pay a contribution. This is because the lobby group knows that, in the absence of an enforcement mechanism, its desired policy will be proposed by a political party only if it is incentive compatible. This requires that the political party is adequately compensated for the utility loss it suffers from altering its policies. A party that spends a lower proportion of its political contributions on campaigning suffers a greater loss when its policy deviates from the vote maximising strategy. To compensate for this, the lobby group is compelled to increase its political contributions. We demonstrate that the likelihood of sequential effects dominating depends on the persuasiveness of the lobbying campaign on the electorate. If the pool of undecided voters is large enough, the sequential effects could possibly dominate. Put simply, this result implies that misappropriation by politicians pays dividends in the form of contributions.

We then extend our model to the case of multiple interest groups approaching one political party. This case is shown to successfully accommodate a collective policy on a public investment project like a park or an airport. It also fits well with a situation in which politicians care about more than one issue at a time. We prove that under such a collective policy, policy beneficial groups may either see themselves paying different amounts of contributions to the political party or enjoying different levels of utility depending on their policy standpoints. In addition, all results obtained from a single lobby group case can also be applied to this multiple group setting.

We also investigate a two-sided lobbying environment in which each political party is aligned with one or more lobby groups. It is shown that each side will be able to reach an agreement on a policy platform. Moreover, that policy is unique. When one party allocates a bigger proportion of its
contributions to campaigning, its aligned lobby group(s) will have a greater voice on the desired platform. In the meantime, the political rival reacts by pivoting towards the median to consolidate votes among informed voters. Thus, policy positions become strategic substitutes. This implies that asymmetry in lobbying prowess may help lessen policy distortions. The intuition is straightforward. One party uses paid campaign contributions to garner more votes and seats in the legislature since it is aligned to a strong lobbyist with ‘deep pockets’. The other party can minimise its loss by pivoting to a more centrist policy platform since it lacks the campaign funds.

In examining the impact of lobbying on electoral competition and policy making process, many studies have identified reasons why campaign contributions are given and how they are used. In particular, as summarised in Bombardini and Trebbi (2011), contributions are given in order to affect the policy choice of an incumbent government (Grossman and Helpman (1999), Polk et al. (2014)) to influence the platform of political candidates (Grossman and Helpman (1996)), to increase the chance of winning election of a candidate (Grossman and Helpman (1996)) or to buy access (Austen-Smith (1995)). For politicians, those contributions are useful because campaign spending can be used to inform voters of a candidate’s position (Austen-Smith (1987)) or to convince them of the candidate’s quality (Coate (2004)). The model of lobbying presented in this paper is in line with the view of ‘money for policy favours’ of campaign funds, especially applied to the strand of menu-auction models (Baron (1994), Grossman and Helpman (1996)) in which special interest groups use campaign contributions to influence decision-makers.\footnote{Although not study here, this paper recognises other viewpoints that consider lobbying either as ‘information transmission’ (e.g. Schnakenberg (2017), Cotton (2009, 2012)), ‘interactions of contributions and information’ (e.g. Bennedsen and Feldmann (2006)), ‘limited or costly access to policy-makers’ (e.g. Austen-Smith (1995), Lohmann (1995) and Cotton (2009, 2012)), or ‘legislative subsidies’ (e.g. Hall and Deardorff (2006)).}

In particular, it pays attention to the case interest groups donating campaign funds to political candidates running for office. Notably, it makes an important departure from the current literature by assuming that these politicians do not use all of these funds for their election campaign. Instead, they embezzle some of these contributions for their private purpose. By doing so, this paper aims to capture two types of distortion to campaign contributions, a legal channel through which interest groups affect policy choices in many democracies. The first distortion is related to an inherent motive of interest groups: money offers are conditional upon policy favours. The second distortion is concerned with emerging stylised facts in world politics: policymakers embezzle campaign funds.\footnote{Among widely publicised cases is a US experience in which Republican Florida Senator}
examine and discussed in the literature, the second distortion is rather new. Hence, there is a strong need in investigating this type of distortion, especially on its impact, either independently, or in interaction with the other form of distortion, on electoral competition and the policy-making process. This paper, therefore, helps to fill this important literature gap.

The paper is organised as follows. In the next section, we outline the basic structure of the model and disclose the channels through which different forms of misuse of campaign contributions affect policy-making. Section 3 investigates equilibrium policy positions and factors affecting this equilibrium such as popularity bias and the degree of embezzlement. The robustness of results is obtained through various settings ranging from a single lobby group to a multiple lobbying context. Section 4 extends the model to cover the case of two-sided lobbying. Section 5 discusses alternative assumptions regarding contexts of lobbying and their implications on the model’s results together with some concluding remarks concerning future research possibilities.

2 The Model

We examine a jurisdiction with two financially constrained political parties, one interest group and a fixed continuum of voters. The interactions between these agents are modelled as a four-staged game. In Stage 1, the special interest group offers to each party a menu of contributions specifying a monetary contribution for each possible policy position that the party might adopt. In Stage 2, upon observing the menus of contributions offered, the parties choose their policy positions. In Stage 3, the special interest group pays the promised contributions given the positions adopted by the parties. Finally, in Stage 4, the election is held in which the voting result is probabilistic.

The following is a description of the details of the election game and the agents’ strategic behaviours. We start our description with the political parties first by denoting the two political parties as $A$ and $B$ respectively. For simplicity, assume that each party consists of only one politician. Hence, throughout this paper, we use the words ‘party’ and ‘politician’ interchangeably. The politicians in both parties are self-interested. The utility function of the representative politician from party $j = (A, B)$ is given by

$$G_j = s [(1 - \theta_j)C_j + R] + (1 - s) [(1 - \theta_j)C_j + L],$$

(1)

Marco Rubio was accused of using his party’s campaign fund to cover personal expenses (The Guardian, 06 June 2015). In Australia, former Victorian Liberal Party state director Damien Mantach was alleged to have embezzled around $1.5 million of election campaign funds (ABC News, 21 August 2015).
where $s$ is the probability that party $j$ wins the election, $C_j \geq 0$ is the campaign contribution received by party $j$ from the special interest group, $\theta_j \in [0, 1]$ defines the proportion of contribution used for campaigning in the election by the relevant party, hence, $(1 - \theta_j)C_j$ is the proportion of campaign contributions diverted for personal use and other non-electoral purposes, and $R$ and $L$ represent the pay-offs when the party wins or loses the election, respectively. It is assumed that $R > L$ so that politicians obtain greater utility from winning an election. This is because of the higher salary and benefits which accrue to the winner and the ‘ego rents’ of holding office. In this formulation, the politicians obtain utility not only from election outcome but also from diverting campaign funds to personal consumption and other uses. To simplify notations, we normalise $L$ to 0. Hence, party $j$’s utility function becomes

$$G_j = (1 - \theta_j)C_j + sR,$$

with $R > 0$ denoting the election winning premium.\(^4\)

Following Grossman and Helpman (1996), we assume that the parties compete on a pliable policy issue, denoted $P$. The pliable issue consists of policy matters such as the possibility of pleasing the special interest group in order to attract campaign finance, etc. Each party’s announced policy position on the pliable issue is represented by $P_j$ ($j = A, B$). The departure from $P_j$, therefore, imposes a quadratic loss.

There are two types of voters in the electorate, the informed and the uninformed ones. Without loss of generality, let the mass of voters be unity. Then, there is a fraction $(1 - \alpha)$ of voters called ‘informed voters’, who have full knowledge of the impact of policies on their welfare and, therefore, have well defined preferences over the pliable policy. We assume that their preferences are given by

$$u_i = -(P_i - P_j)^2,$$

where $P_i$ is the preferred policy of an informed voter $i$, and $P_j$, as defined above, is the pliable policy issue of party $j$ ($j = A, B$).

Assume that the parties cannot directly observe $P_i$, but know that the representative voter’s relative preference follows a uniform distribution over the range below, as per Grossman and Helpman (1996):

$$u_i(P_A) - u_i(P_B) \sim \left[ \frac{1}{2d} - \frac{b}{d}, \frac{1}{2d} - \frac{b}{d} \right].$$

\(^4\)In this paper, $\theta_j$ is an exogenous parameter determined by the candidate’s type. Clearly, endogenising $\theta_j$ (i.e. letting it be chosen optimally by the candidate) will always result in $\theta_j = 0$. This is because the candidate’s utility function given in (2) is decreasing in $\theta_j$. 

6
where \( d > 0 \) is a measure of the diversities of views about the parties and \( b \) represents the inherent ideological bias of voters for party \( j \). The voter will vote for party \( A \) if and only if \( u_i(P_A) - u_i(P_B) \geq 0 \). Both parties will perceive a probability of \( F[u_i(P_A) - u_i(P_B)] \) that voter \( i \) will vote for party \( A \). Applying the law of large number, the share of informed voters casting a vote for party \( A \) will be equal to \( \frac{1}{n_I} \int_{i \in I} F[u_i(P_A) - u_i(P_B)] \, di \), where \( I \) is the set of informed electorates and \( n_I \) denotes the mass of those individuals. Using the utility function described in (3), we have

\[
u_i(P_A) - u_i(P_B) = -\left(\frac{1}{2} - \frac{b}{d}\right) \left(\frac{b}{2d} - \frac{b}{2}\right) + \left(\frac{p_B}{2d} - \frac{p_B}{2}\right).
\]

Now we simplify notations by letting \( \rho_1 = -\left(\frac{1}{2} - \frac{b}{d}\right) \) and \( \rho_2 = \frac{1}{2} - \frac{b}{d} \). By assumption, \( u_i(P_A) - u_i(P_B) \) follows a uniform distribution over \([\rho_1; \rho_2]\) which implies the following:

\[
\begin{cases} \frac{1}{\rho_2 - \rho_1}, & \rho_1 \leq u_i(P_A) - u_i(P_B) \leq \rho_2 \\ 0, & \text{elsewhere} \end{cases}
\]

where \( f(\cdot) \) denotes that probability density function. This gives the cumulative distribution function as follows:

\[
F[u_i(P_A) - u_i(P_B)] = \begin{cases} \frac{u_i(P_A) - u_i(P_B)}{\rho_2 - \rho_1}, & \rho_1 \leq u_i(P_A) - u_i(P_B) \leq \rho_2 \\ 0, & \text{elsewhere} \end{cases}
\]

Plugging in the values for \( \rho_1 \) and \( \rho_2 \) we get:

\[
F[u_i(P_A) - u_i(P_B)] = \frac{1}{2} + b + d[u_i(P_A) - u_i(P_B)],
\]

for \( \rho_1 \leq u_i(P_A) - u_i(P_B) \leq \rho_2 \). In this formulation, similar to Grossman and Helpman (1996), \( b \) is interpreted as informed voters’ bias towards party \( A \) if \( b > 0 \) (or towards party \( B \) if \( b < 0 \)). When \( b = 0 \), both parties are equally popular among those informed voters.

There is a fraction \( \alpha \) of voters called 'uninformed voters' in the electorate who are unable to assess the impact of the pliable policy issues on their welfare. They can be persuaded to vote for a party through campaign advertisements. The party which spends more on campaign advertisements captures a greater number of their votes. As in Grossman and Helpman (1996), the proportion of this electorate who vote for party \( A \) is given by

\[
\frac{1}{2} + b + h (\theta_A C_A - \theta_B C_B),
\]

where \( h > 0 \) measures the effectiveness of the advertisement campaign.
Combining (4) and (5), the proportion of individuals who vote for party A will be

\[ s = \frac{\alpha h}{2} \left( \theta_A C_A - \theta_B C_B \right) + (1 - \alpha) \int d[U(P_A) - U(P_B)], \]  

(6)

where \( U(P) = \frac{1}{n} \sum_{i \in I} u_i(P) \) is the average welfare of all informed voters. In the absence of any lobbying activity, the optimal policy for each party is to support the policy that best serves the average informed voter. Such a voter expects the midpoint of the policy positions by the candidates, \( \frac{P_A + P_B}{2} \), as the implemented policy. As a result, the number of voters supporting party A in this legislature will be given by:

\[ s = \frac{1}{2} + b, \]  

(7)

In this paper, it is assumed that the eventually adopted policy will be the one that is determined by the winning party. Normally, that is the policy platform that the winning party announces before the election.\(^5\)

The special interest lobby group has a well-defined preference over its pliable policy issue. It seeks to influence the policy outcome by making campaign contributions to the parties in order to induce them to adopt the policy platform that better serves its needs. Let \( C_j \) denote the campaign contributions paid by the lobby group to party \( j \) (\( j = A, B \)). The expected utility of the lobby group from a policy is given by

\[ u_L = -s(P_A - P_L)^2 - (1 - s)(P_B - P_L)^2 - C_A - C_B, \]  

(8)

where \( P_L \) is the lobby group’s ideal policy.

Having outlined the general structure of the model, we now derive the equilibrium level of contributions paid to each party by the lobby group. In the absence of lobbying, party A would choose a pliable policy \( \hat{P}_A \) to maximise its utility, as defined in (1). Thus,

\[ \hat{P}_A = \arg \max \left( sR \right). \]  

\(^5\)It is assumed that there is no time inconsistency problem in this model, i.e. the parties do not renege on their previously announced policies. This is because parties have an incentive to develop clear and persistent reputations with voters in order to win future elections. However, the parties can renege on their promises with special interest groups regarding the use of campaign funds due to an absence of a legally binding contract between the two sides. This allows the introduction of a moral hazard problem for the politicians in terms of campaign fund embezzlement. According to Kroszner and Stratmann (1998) and Groll and Ellis (2017), compliance in agreements between politicians and special interest groups can be obtained through repeated lobbying. For the purpose of investigating the consequence of politicians’ misuse of campaign funds on electoral competition and policymaking, we do not model repeated lobbying in this paper. This also helps us keep our model more tractable.
Because the objective function is increasing in \( s \), party A will choose the policy that helps it attract the largest number of informed voters. In this case, the number of informed voters that party A attracts in the legislature is the one that is calculated in (7) so \( \hat{s} = \frac{1}{2} + b \). Assume that there is a continuum of policy range in \([-1,1]\) in which party A chooses a policy position \( \hat{P}_A \in [0,1] \) and party B chooses \( P_B \in [-1,0] \). Equilibrium condition dictates that the optimal policy chosen by party A is \( \hat{P}_A = 0^+ \) (on the right side of the median value 0) while that of party B is \( \hat{P}_B = 0^- \) (on the left side of the median value 0). This is also the policy platform that an average informed voter expects the parties to implement.

Now suppose that the lobby group offers party A a contribution to conduct a policy platform \( P_A \) which is more closely aligned to the lobby group’s interest. Party A will have an incentive to alter its policy (i.e. from \( \hat{P}_A \) to \( P_A \)) only if the contribution it receives from the lobby group, \( C_A \) is sufficient to compensate for its policy variation. Thus, contribution to party A must satisfy \( G \geq \hat{G} \). This implies

\[
(1 - \theta_A)C_A + R[\alpha h \theta_A C_A + (1 - \alpha)d(U(P_A) - U(\hat{P}_A))] \geq 0. \tag{10}
\]

In this formulation, the direct cost of campaigning is \( \theta_A C_A \) while its benefit is \( R\alpha h \theta_A C_A \). Party A will only conduct an election campaign if the benefit outweighs the cost of such an action. Hence, we state the following assumption:

**Assumption 2.1.** Assume that \( R, \alpha \) and \( h \) satisfy the following condition

\[
R \alpha h > 1. \tag{11}
\]

This assumption imposes the reasonable condition for a party to conduct an election campaign. Clearly, \( R\alpha h \) is the marginal benefit of campaigning while 1 is the marginal cost of that action. If this condition is not satisfied, the optimal policy for the political party under consideration is simply not to launch any election campaign despite taking the financial contribution from the interest group. Rearranging this result, one obtains \( \alpha > \frac{1}{Rh} \). This indicates that the fraction of uninformed voters must be large enough for the campaigning program to be worth taking place.

Now we simplify (10) to get

\[
C_A \geq \frac{R(1 - \alpha)d[U(\hat{P}_A) - U(P_A)]}{1 - \theta_A + R\theta_A \alpha h}. \tag{12}
\]

As can be seen from (8), the lobby group’s utility is declining in the contribution paid to each party. The lobby group, therefore, maximises its utility
by offering party $A$ the minimum amount necessary to induce it to modify its implemented policy to a $P_A > 0$ from its preferred position $P_A = 0$. Thus, in equilibrium, contributions must satisfy (12) with equality. Given that $U(P_A) = -P_A^2$ and $U(\hat{P}_A) = 0$ where 0 denotes the average ideological preference of informed voters, we have:

$$C_A = \frac{R(1 - \alpha)dP_A^2}{1 - \theta_A + R\theta_Aa}.$$  \hfill (13)

Clearly, the amount of contribution $C_A \geq 0$ for $P_A \geq 0$. An analogous condition holds for party $B$ so that

$$C_B = \frac{R(1 - \alpha)dP_B^2}{1 - \theta_B + R\theta_Ba}.$$  \hfill (14)

Here, we also have $C_B \geq 0$ for $P_B \leq 0$ (by assumption). These equations reflect that in the bargaining process with the political parties, the lobby group has to contribute more if it wants the parties to cater more for its desired policy. Substituting the result for party $A$ into (1) yields:

$$G_A = \hat{G} = R\left(\frac{1}{2} + b\right).$$

A similar result is obtained for party $B$ (i.e. $G_B = R\left(\frac{1}{2} - b\right)$). The result indicates that, under this current setting, each party’s optimal utility level is always guaranteed when it enters into an agreement with the interest group. This guaranteed level is independent of the party’s choice of $P$ at the optimum. Technically, this allows the interest group to make the optimal policy of their choice as soon as it fully compensates the political parties for their policy variations. In other words, the lobby group can offer the parties its contributions contingent on policy platforms.

3 Equilibrium Policy Positions

Having obtained the equilibrium level of contributions, we now solve for the policy positions. As noted by Grossman and Helpman (1996), the lobby group can induce the political parties to announce any feasible policy platform so long as the contributions adequately compensate them for the welfare loss due to the departure from their preferred policy positions. Thus, the policy platform announced by each party will maximise the lobby group’s payoffs, subject to the constraint that each party is adequately compensated
for the utility loss of varying the announced policy from its preferred position. Hence, the lobby group solves the following problem:

\[ \max_{P_A, P_B} u_L = -s(P_A - P_L)^2 - (1 - s)(P_B - P_L)^2 - C_A - C_B, \]  

subject to the constraints in (13) and (14) respectively. This is a standard utility maximisation with two choice variables, \( P_A \) and \( P_B \). This problem can be solved by taking derivative of the utility function with respect to each choice variable. However, before proceeding into these first order necessary conditions, it is important to have the following observations:

**Proposition 3.1.** The lobby group will contribute to only one political party. The chosen party will be the one whose policy is more closely aligned to the lobby group’s ideal policy.

**PROOF:** see Appendix.

Proposition 3.1 says that the parties are distinguished by different ideological viewpoints which are shaped by a long history or tradition. Knowing that it is not possible to persuade the parties to switch sides of their policy standpoints (for example from the \([-1,0]\) region to \([0,1]\) region or vice versa) due to extremely high costs or high ideological barriers set by them, the interest group will need to identify which party that has more closely aligned standpoint with its ideal policy. If the interest group decides to undertake the lobbying activity, it will only make a financial contribution to that associated party.

The result that a lobby group will only make financial contribution to one political party is rather standard. It has been pointed out and discussed in Austen-Smith (1987), Baron (1988) and Hall and Deardorff (2006) despite using different modelling frameworks. In particular, according to Baron (1988), there exists empirical evidence showing that most political action committees contribute to only one candidate. This result greatly simplifies the mathematical work needed for executing the maximisation problem stated in (15).

**Proposition 3.2.** Between the two policies of the same distance to its ideal platform, the lobby group will only push for the one that is closer to the median.

**PROOF:** see Appendix.

\footnote{Hall and Deardorff (2006) point out that if lobbying is a type of legislative subsidy, lobby groups will ‘only lobby their allies’ and will never lobby ‘their enemies’.}
An important result standing out from Proposition 3.2 is that \(|P| \leq |P_L|\) meaning the interest group will never over-lobby the political party as doing so is not utility maximising. As shown below, this result is needed for having a unique equilibrium policy outcome which is the product of the bargaining process between the party and the interest group.

For simplicity, now suppose that party A is the chosen party by the lobby group due to its more closely aligned policy. This means that party B will not need to vary its policy and, hence, receives no contribution. In other words, we can set \(P_B = 0\) and \(C_B = 0\). In addition, as per Proposition 4.2, \(P_A \leq P_L\). The maximisation problem in (15) can now be rewritten as:

\[
\max_{P_A} u_L = -s(P_A - P_L)^2 - (1 - s)P_L^2 - C_A, \tag{16}
\]

subject to the constraint in (13). The proportion of voters in support of party A described in (6) will now collapse to:

\[
s = \frac{1}{2} + b + \alpha h \theta_A C_A - (1 - \alpha) d P_A^2 = \frac{1}{2} + b - \Omega_A P_A^2 \tag{17}
\]

where

\[
\Omega_A = [(1 - \alpha) d - \alpha h \theta_A \Phi_A] = \frac{(1 - \alpha) d (1 - \theta_A)}{1 - \theta_A + R \theta_A \alpha h}
\]

and

\[
\Phi_A = \frac{R (1 - \alpha) d}{1 - \theta_A + R \theta_A \alpha h}.
\]

As \(0 \leq s \leq 1\), to assure that this condition always hold, we need to make the following assumption:

**Assumption 3.3.** Parameters are such that

\[0 \leq \Omega_A \leq \frac{1}{2} + b \leq 1.\]

Clearly, the condition \(0 \leq \Omega_A\) is automatically satisfied given \(\alpha \in [0, 1], \theta_A \in [0, 1], d > 0\) and \(R \alpha h > 1\) (as per Assumption 3.1). In order to have \(0 \leq s \leq 1\) for \(\forall P_A \in [0, 1]\), we impose \(\Omega_A \leq \frac{1}{2} + b \leq 1\). Note that while \(\alpha h \theta_A C_A\) reflects the gain in uninformed voters’ votes of party A, \((1 - \alpha) d P_A^2\) captures its loss from the informed voters’ votes due to policy variation. Hence, \(\Omega_A\) is the net marginal change in votes of party A once it deviates its policy platform from the median value. Under these conditions, we can state the result below:

**Lemma 3.4.** When there is only one lobby group taking part in the election game, there exists a unique equilibrium policy platform that is mutually agreeable by the chosen political party and the lobby group.
This lemma indicates that in this equilibrium, the lobby group reaches an agreement with the political party on an implemented policy, $P_A^*$. This policy maximises the lobby group’s interest while leaving the political party no worse off thanks to the lobby group’s financial payment. This amount is equal to $C_A^* = \Phi_A(P_A^*)^2$.

**Proposition 3.5.** Other things equal, as the bias in the electorate for the chosen party increases, the lobby group will find it easier to negotiate with the political party on a policy platform that is closer to its desired one.

**PROOF:** see Appendix.

The intuition behind Proposition 3.5 is the following. Once the party in action becomes more popular, it has more confidence in winning votes. As a result, it can tailor its policy more towards the desired platform of the interest group. This is because the extra popularity allows it to balance between the marginal loss of informed votes resulted from further deviating from its median voting position with the marginal gain in the lobby group’s contribution through two different channels: (i) additional uninformed votes by extra spending on election campaign; and (ii) more financial resources for personal use. In the meantime, the lobby group finds it more beneficial to induce this more popular party to adopt a platform closer to its preferred position. This is simply because, *ceteris paribus*, contributions given to the more popular party are more productive in buying votes, so the opportunity cost to the political party of a policy deviation is lower.

**Proposition 3.6.** Other things equal, the equilibrium policy platform does not always increase with the lobby group’s ideal policy. In particular, there is a hump-shaped relationship between these two variables.

**PROOF:** see Appendix.

The result set out in this proposition can be explained as follows. Differences between the lobby group’s ideal policy platform and that of the party have non-linear consequences. When the lobby group’s ideal policy is close to the median policy, the political party can accommodate a move away from the median because the loss of informed votes is relatively small. However, when the lobby group’s ideal policy is sufficiently far away from the median policy, the electoral cost is higher so the political party has an incentive to move back to the median to attract a higher share of informed votes.

---

7 In a different context, Bombardini and Trebbi (2011) investigate the relationship between the size of interest groups in terms of voter representation and the interest group’s campaign contributions to politicians. They uncover a robust hump-shaped relationship between the voting share of an interest group and its contributions to a legislator.
Proposition 3.7. When misappropriation of campaign contributions, as measured by the degree of embezzlement $1 - \theta$, rises the lobby group has less influence on the announced policy platform of the chosen party.

PROOF: see Appendix.

Proposition 3.7 explores the manner in which policy platforms vary with changes in the degree of embezzlement. Intuitively, a party that diverts a higher proportion of campaign contributions towards personal use, spends a lower proportion of contributions on campaigning. To obtain a previous policy platform, it requires a higher contribution from the interest group. Thus, contributions are less effective in delivering the lobby group’s desired policy platform. Hence, as more funds are diverted for personal use, the lobby group has less influence on the announced policy platform. Paradoxically, this result suggests that in an election context, parties which embezzle more funds will stand on more independent platforms.

Proposition 3.8. Assume that the fraction of uninformed voters is sufficiently large, i.e. $\alpha > \frac{1}{R_h}$. An increase in this fraction will induce the political party to cater more for the interest group’s policy.

PROOF: see Appendix.

The result in this Proposition 3.8 is highly intuitive. A higher proportion of uninformed voters creates a higher incentive for the political party to compete for their votes. As a result, the party is willing to cater more for the lobby’s group policy in return for a greater contribution.

So far, we have assumed that there is only one lobby group. Now we consider the case of multiple interest groups approaching one party. To see how results will change, we continue to assume party A to be the chosen party for the lobby groups. In this setting, the party and interest groups negotiate over a collective policy in which such a policy is binding on all interest groups.\(^8\) An example of this policy type is the scale of a public good such as building a park or an airport in a local neighbourhood. Choosing a location for a public good could be a tricky decision for the political party. While building a new park certainly adds more value to the local area, having a new airport seems to do more harm than benefit to its neighbourhood due to the noise it creates. However, a decision must be made on the location of that public good. Clearly, it is impossible for the party to choose a policy that maximises every individual interest group’s utility. Another example is the situation in which the party cares about more than one issue at a

---

\(^8\)Levine and Modica (2017) also study the issue of multiple interest group lobbying a politician but in a different environment. In their model, interest groups compete for policy agenda and the group with the winning bid becomes the agenda setter.
time. For instance, among the interest groups, one may value environmental quality more than output growth. Another may put the latter ahead of the former in their action plan. Each of them might dislike the policy objective of the other. Hence, in each of these contexts, we assume that the political party will maximise the joint utility of all groups. Under this setting, we can state the obtained result in the following proposition:

**Proposition 3.9.** When there are many interest groups lobbying one political party, there is a unique equilibrium policy platform that is agreed by the party and all the lobby groups. Under this equilibrium policy platform, lobby groups either contribute different amounts or enjoy different levels of utility.

**PROOF:** see Appendix.

Basically, Proposition 3.9 says that results obtained can be extended to the case of $N \geq 2$ interest groups negotiating with one political party over an implementable policy platform while leaving the other political party untouched. The key difference between this multiple group case and the previous single group case is that the party will take the average policy standpoint of all the groups, rather than that of a single group, into consideration.$^9$ However, under this mutually agreeable policy platform, the lobby groups, due to their different ideal platforms, will be treated differently either in terms of contributing amounts or levels of utility. In particular, if they want to assure the same level of utility across groups, they will have to pay unequal amounts of contributions. If they decide to pay an equal amount of contribution to the political party, they will see the levels of obtained utility vary across the board.

### 4 Two-sided Lobbying

Now we consider the case of two interest groups with each lobbying one political party. Assume that a group with an ideal policy $q_A$ approaches party $A$ from the right and another group with an ideal policy $q_B$ approaches party $B$ from the left. By this assumption, $q_A > 0$ and $q_B < 0$. We limit our attention to the case that each lobby group is only associated with its closely aligned party but not the other. This is because lobbying the other party to switch its policy standpoint is not possible due to extremely high costs or

$^9$Because we assume that the joint utility function is a simple sum of all individual groups’ utility, the average policy standpoint is a simple average of all groups’ policy platforms. It can be easily verified that if the joint utility takes a weighted average form, the ultimate policy standpoint will also be of the weighted average form. However, this assumption does not change the qualitative result of our model.
the high ideological barrier set by the party. As a result, each lobby group will only negotiate with its closely related party for a policy platform that maximises its utility upon paying its adequate contributions to the party. In doing so, each lobby group takes actions of the other lobby group and the other party as given.

Under the above described setting, we have the following result:

**Lemma 4.1.** In case of two-sided lobbying, each lobby group will be able to reach an agreement with its associated political party on an implemented policy platform. For each side of coalition, such an agreement is unique.

PROOF: see Appendix.

In the next step, we will check the potential impact of the parties’ embezzlement degrees on the policy platforms conducted. The results obtained can be summarised in the proposition below:

**Proposition 4.2.** Other things equal, as one party allocates a greater proportion of its lobby group’s contribution to campaigning, its contributing lobby group negotiates for a policy closer to its desired platform. Meanwhile, rivals (the other political party and its partnered group) are compelled to pivot closer to the median policy position.

PROOF: see Appendix.

This proposition reveals that campaign intensities are strategic substitutes, under these asymmetric conditions. When one party competes more aggressively by allocating a greater proportion of its funds to campaigning, its rivals become more cautious on the policy platform it announces. Intuitively, when the party under consideration campaigns intensively, its rival’s campaign spending is less productive in terms of acquiring votes from the uninformed electorate. Moving closer to the median to consolidate the number of votes among informed voters thereby becomes the better strategic move for the rival. It, therefore, announces a policy platform which is closer to the ideal one of the average informed voter. By contrast, the party diverts a smaller portion of its funds to personal use and adopts a policy which is closer to the preferences of the lobby group. These results indicate that electoral competition may lessen policy distortions. With greater competition in elections there is less incentive to propose policies that promote the views of special interest groups. However, this in turn induces higher contributions from the interest groups (i.e. more intense electoral competition) may help lessen policy distortions, but creates incentives to extract rents from lobby groups.\(^\text{10}\)

\(^{10}\)The results obtained in this proposition can somehow be supported by observations in
5 Conclusion and Extension

This paper constructs a stylised model of monetary contributions in exchange for policy favours in an electoral competition framework. In particular, lobby groups seek to influence financially constrained policy-makers with the help of campaign contributions. Two key assumptions are embedded in the model: (i) voters participate in the election; and (ii) the election is competitive. Voters are assumed to be rational in the sense that they vote for the politician (and policy) of their best interest given the available information. When selling favours, politicians use an all-pay auction mechanism to attract contributions from special interest groups and vote in favour of their policy standpoints in return. Due to informational asymmetry between lobby groups and voters, these contributions are expected to be spent on persuading uninformed voters in the election. As this informational asymmetry becomes larger, the significance of the campaign contributions become larger.

However, not all contributions are spent on electoral campaigning as part of the amount may be used by the politicians to pursue their private rents. In this framework, special interest groups are faced with an uncertain environment. They make contributions to the party or politician of their choice while being uncertain about the outcome of their actions. In particular, they are not sure if the politician will use their contributions for electoral competition to increase the likelihood of electoral success. If not, the politician may fail to deliver the promised policy due to not being (re)elected. This uncertainty narrows the scope at which special interest groups exert influence on the policy-making process. In the meantime, politicians may earn some non-electoral or non-office rents from embezzlement of funding contributions and voters are disadvantaged by having less information on announced electoral

US politics. When working as the Secretary of State in the Obama Administration during 2009-2013, Hillary Clinton was a strong supporter of the Trans-Pacific Partnership, a free trade agreement among 12 countries in the Pacific Rim. However, in her second presidential run in 2016, facing with an increasing protectionism from American voters, especially from industrial, labour and environmental groups, she changed her stance on this multilateral trade deal saying that it did not meet her standard (Business Insider, 10 October 2016).

Another observation is the gun control issue. According to Vox (5 October 2017), every year, the National Rifle Association (NRA) donates millions of dollars to Republican lawmakers to block proposed gun control legislation (its spending figure for the 2016 election was $14 million - CBS News, 20 June 2016). In 2013, the Senate, despite being controlled by the Democrats, failed to pass the so-called Manchin-Toomey gun control amendment bill due to the lack of support from four Democrats, those who came from rural states with strong gun cultures and were seeking re-election in 2014 (Washington Post, 17 April 2013). Perhaps, they did not want to disappoint the majority of their voters back home.
platforms due to less funds devoted to the electoral campaign. In equilibrium, a special interest group donates only to one party and the alternative use of campaign funds for private rents somehow discounts its policy influence through contributions.

We obtained several interesting and important results that can be summarised as follows. Firstly, we showed that in a single lobby group case, the lobbying activity is only one-sided and there exists a unique agreement between the lobby group and its aligned political party. Higher degrees of misappropriation of campaign contributions measured in terms of embezzlement level will result in a less favourable position for the lobby group. More specifically, the lobby group is required to pay higher contributions to the parties and the parties divert a greater proportion of these contributions for their personal use. Secondly, we generalised the model to the case of multiple interest groups negotiating with one political party over a policy platform. We proved that in order to implement a collective policy, the political party may need to demand different contributions from the lobby groups or satisfy them with different levels of utility. Finally, we considered the case of two-sided lobbying in which each side of coalition is characterised by interest groups negotiating with their sole political party. We have shown that each side will be able to reach an agreement on the implemented policy platform and that such an agreement is unique. Moreover, as one party allocates a greater proportion of its contribution to campaigning, its partnered lobby group(s) will have an incentive to negotiate for a policy closer to the desired platform. In the meantime, its rivals take a more cautious strategic approach. This means that while electoral competition may help reduce policy distortions, it creates incentives for the party to extract rents from lobby groups.\(^\text{11}\)

Despite being a legalised channel and becoming more and more popular in politics of representative democracies, campaign contributions can contain a significant amount of distortion especially when they are (i) made in exchange for policy favours, and (ii) embezzled by policy-makers. Between these two types of distortion, while the former has been well examined in the literature, the latter is rather new and deserves further investigation as a better understanding of this issue will lead to better regulation. To reduce potential distortions of campaign funds, several remedies could be considered such as setting a contribution limit (e.g. Stratmann (2006a,b), Dahm and Porteiro (2008); Cotton (2009, 2012), Koppl-Turyna (2014)), setting a

\(^{11}\)Several theoretical predictions here can be tested using US data developed by Lake (2015) that decomposes representative-specific contributions across issues and issue-specific lobbying expenditures across representatives.
contribution tax (e.g. Cotton (2009)) or both (e.g. Cotton (2009, 2012)). Introducing these measures into the current setting will be interesting but should not change the main results on special interest groups’ strategic behaviours. This is because given the specified utility functions for the party and the lobby group in (3) and (8) respectively, the party still sells policy through a menu auction in which each interest group provides a contribution schedule that assigns a payment to each possible policy choice. The policy choice is still made by the well informed politician upon observing special interest groups’ submissions. In that respect, both a limit and a tax do not further distort the politician’s information.

However, such measures may have some implications on gross lobbying competition as measured by the total amount of money or financial resources spent on electoral competition. Clearly, in the case of a limit, if the limit is binding, total contributions made by lobby groups may be less than the utility maximising amounts. Nevertheless, these amounts should stay the same if the limit is not binding. In the case of a tax, if the politician benefits from this tax revenue, the tax would play a same role as the embezzlement in the current analysis because they both function as a transfer from the lobby group to the politician. This means that total contributions made by interest groups would not change. They would not change even if the tax revenue is given towards some public good as assumed in Cotton (2009). This is because such a tax does not distort the interest groups’ incentives to contribute to the parties’ election campaign.

Although welfare consequences from a planner’s perspective are not considered in this paper, an extension to include this trajectory into the current analysis is possible. Clearly, campaign spending has two opposing effects on uninformed voters’ welfare. Specifically, this spending helps these voters learn better about the politicians they will vote for in an election, which is good for them. However, it also biases the policy choice towards interest groups’ preferences and, hence, away from those of the electorate, which is not good for those voters. In order to conduct a thorough welfare analysis, it is required to have a well-defined social welfare function in which the social planner takes into account the partial welfare of all agents in the society, i.e. the politicians, the special interest groups and the voters. Depending on the shape of this welfare function as well as the weights allocated by the social planner to each player group reflecting his priority, there will be different results on the impact of lobbying on the policy-making process. It will also result in different outcomes on the way in which embezzlement may help limit policy favours (or distortions) and the scope of the trade-off between less policy favour for special interest groups (i.e. a social benefit) and less information for uninformed voters (i.e. a social cost) triggered by this em-
bezzlement.\textsuperscript{12} All these suggest an exciting and promising future research agenda.

**Acknowledgment**

We would like to thank Valerie Caines, Adrian Cheung, Brenton Fiedler, Editor Stratmann of this journal, two anonymous referees and seminar participants at Flinders University for valuable comments and suggestions on earlier versions of this paper. The usual disclaimer applies.

**Funding**

This research did not receive any specific grant from funding agencies in the public, commercial or not-for-profit sectors.

**References**


\textsuperscript{12}Within the current scope of this paper and to make our model tractable, we do not consider these issues analytically here. Rather, we reserve them for our future research. We are grateful to an anonymous referee for this suggestion.


Appendix

Proof of Proposition 3.1

By assumption, the policy range of party A is [0,1] and that of party B is [-1,0]. In the absence of lobbying, both parties will converge to the median policy 0. Assume that there is an interest group with its associated ideal policy of \( P_L > 0 \). It is straightforward to see that this interest group will only need to lobby party A which has a more closely aligned policy because it cannot twist the ideological viewpoint of party B. A symmetric argument applies for the case \( P_L < 0 \).

Proof of Proposition 3.2

In the absence of lobbying, party A will stay at the median policy position \( \hat{P}_A = 0 \). If the special interest group with an ideal policy \( P_L > 0 \) wants to lobby the party then it will maximise its utility by negotiating for the policy platform that requires the least cost among those equally satisfactory ones. Note that the first component of the utility function takes a quadratic form, \(-(P - P_L)^2\). As a result, there are always two values \( P_1 \) and \( P_2 \) such that \( P_1 < P_L < P_2 \) and \(-(P_1 - P_L)^2 = -(P_2 - P_L)^2\). Between these two policy values, policy \( P_1 \) requires a lower contribution cost than policy \( P_2 \) (because at \( P_1 \) there is less policy variation for the political party than at \( P_2 \)) meaning it will be chosen ahead of its counterpart from the utility maximisation point of view.

Proof of Lemma 3.4

The first order necessary condition for the lobby group’s utility maximisation is:

\[
\frac{\partial u_L}{\partial P_A} = -\frac{\partial s}{\partial P_A} (P_A - P_L)^2 - 2s(P_A - P_L) + P_L^2 \frac{\partial s}{\partial P_A} - \frac{\partial C_A}{\partial P_A} = 0 \quad (18)
\]

From (13) and (17) we have:

\[
\frac{\partial C_A}{\partial P_A} = 2\Phi_A P_A
\]

\[
\frac{\partial s}{\partial P_A} = -2P_A \Omega_A
\]

Plugging these into the above first order condition and simplifying yields:

\[
\frac{\partial u_L}{\partial P_A} = \Omega_A P_A^3 - 2\Omega_A P_L P_A^2 - (s + \Phi_A) P_A + s P_L = 0 \quad (19)
\]
To check for the second order sufficient condition for a maximum, we differentiate the left hand side (LHS) of this equation with respect to $P_A$ to obtain:

$$\frac{\partial^2 u_L}{\partial P_A^2} = 5\Omega_A P_A (P_A - P_L) - \Omega_A P_L P_A - (s + \Phi_A) < 0 \quad (20)$$

This condition holds as $0 \leq P_A \leq P_L$ as per Proposition 4.2. This means that the second order sufficient condition is satisfied for a maximum. It also means the LHS of the first order condition is decreasing in $P_A$. Given that $P_A \in [0, 1]$, the range of value for the LHS is $[-\Omega_A - \Phi_A, (\frac{1}{2} + b)P_L]$ spanning across a negative to positive region. This implies that there exists a unique solution $P_A^* \in [0, 1]$ that solves the first order condition due to a single crossing property.

**Proof of Proposition 3.5**

Differentiating (19) with respect to $b$ and noting $\frac{\partial s}{\partial b} = 1 - 2\Omega_A P_A \frac{\partial P_A}{\partial b}$, one obtains

$$[5\Omega_A P_A^2 - 6\Omega_A P_L P_A - (s + \Phi_A)] \frac{\partial P_A}{\partial b} = P_A - P_L$$

Because $0 \leq P_A \leq P_L$, the square bracket on the LHS is negative as per (20) and the RHS is also negative. Hence, it must be that $\frac{\partial P_A}{\partial b} > 0$.

**Proof of Proposition 3.6**

Totally differentiating (16), we obtain:

$$\frac{\partial^2 u_L}{\partial P_A^2} \tilde{D} P_A + \frac{\partial^2 u_L}{\partial P_A \partial P_L} \tilde{D} P_L = 0$$

After rearranging this, we get:

$$\frac{\tilde{D} P_A}{\tilde{D} P_L} = -\frac{\partial^2 u_L}{\partial P_A \partial P_L}$$

Given that $\frac{\partial^2 u_L}{\partial P_A^2} < 0$ as in (20), $\text{sign}(\frac{\partial P_A}{\partial P_L}) = \text{sign}(\frac{\partial^2 u_L}{\partial P_A \partial P_L})$. In addition, using (19) then (16), it can be derived that:

$$\frac{\partial^2 u_L}{\partial P_A \partial P_L} = s - 2\Omega_A P_A^2 = \frac{1}{2} + b - 3\Omega_A P_A^2$$

Notice that at the critical value $\tilde{P}_A = \sqrt{\frac{1 + 2b}{6\Omega_A}}$, this term starts to change its sign. Given that $P_A \leq 1$, if $\tilde{P}_A > 1$ then $P_A < \tilde{P}_A$ so $\frac{\partial^2 u_L}{\partial P_A \partial P_L} > 0$ making
\[ \frac{\partial P_A}{\partial P_L} > 0. \] Similarly, when \( P_A < \bar{P}_A \leq 1, \frac{\partial P_A}{\partial P_L} > 0. \) An increase in \( P_L \) leads to an increase in \( P_A \). By contrast, when \( 1 \geq P_A > \bar{P}_A, \frac{\partial^2 u_L}{\partial P_A \partial P_L} < 0 \) resulting in \( \frac{\partial P_A}{\partial P_L} < 0. \) An increase in \( P_L \) entails a fall in \( P_A \). And when \( P_A = \bar{P}_A, \frac{\partial P_A}{\partial P_L} = 0. \) This means that the relationship between the lobby group’s ideal policy and the mutually agreed policy follows a hump-shaped pattern.

**Proof of Proposition 3.7**

Totally differentiating (16) and rearranging we get:

\[ \frac{\hat{D}P_A}{\hat{D}\theta_A} = -\frac{\frac{\partial^2 u_L}{\partial P_A \partial \theta_A}}{\frac{\partial^2 u_L}{\partial^2 P_A}} > 0. \]

We have

\[ \frac{\partial^2 u_L}{\partial P_A \partial \theta_A} = (2P_A^3 - 3P_LP_A^2) \frac{\partial \Omega_A}{\partial \theta_A} - P_A \frac{\partial \Phi_A}{\partial \theta_A} \]

Note that because \( P_A \leq P_L \), the term inside the bracket is negative. In addition,

\[ \frac{\partial \Omega_A}{\partial \theta_A} = \frac{-R\alpha h(1 - \alpha)d}{(1 - \theta_A + R\theta_A \alpha h)^2} < 0 \]

and

\[ \frac{\partial \Phi_A}{\partial \theta_A} = \frac{R(1 - \alpha)d[1 - R\alpha h]}{(1 - \theta_A + R\theta_A \alpha h)^2} < 0 \]

where the second result holds as per Assumption 1. All these indicate \( \frac{\partial^2 u_L}{\partial P_A \partial \theta_A} > 0. \) In addition, \( \frac{\partial^2 u_L}{\partial^2 P_A} < 0 \) as in (20). Thus, we have

\[ \frac{\hat{D}P_A}{\hat{D}\theta_A} = -\frac{\frac{\partial^2 u_L}{\partial P_A \partial \theta_A}}{\frac{\partial^2 u_L}{\partial^2 P_A}} > 0. \]

This implies \( \frac{\partial P_A}{\partial (1-\theta_A)} < 0. \)

**Proof of Proposition 3.8**

Totally differentiating (16) and rearranging gives:

\[ \frac{\hat{D}P_A}{\hat{D}\alpha} = -\frac{\frac{\partial^2 u_L}{\partial P_A \partial \alpha}}{\frac{\partial^2 u_L}{\partial^2 P_A}} \]
From (19) and (17), we have:

$$\frac{\partial^2 u_L}{\partial P_A \partial \alpha} = (2P_A^3 - 3P_L P_A^2) \frac{\partial \Omega_A}{\partial \alpha} - P_A \frac{\partial \Phi_A}{\partial \alpha}$$

Because $P_A \leq P_L$, the term inside the bracket is negative. Moreover,

$$\frac{\partial \Omega_A}{\partial \alpha} = -\frac{d(1 - \theta_A)[1 - \theta_A + R\theta_A h]}{(1 - \theta_A + R\theta_A h)^2} < 0$$

and

$$\frac{\partial \Phi_A}{\partial \alpha} = -\frac{dR[1 - \theta_A + R\theta_A h]}{(1 - \theta_A + R\theta_A h)^2} < 0$$

Plugging these into the above equation we have $\frac{\partial^2 u_L}{\partial P_A \partial \alpha} > 0$. In addition, $\frac{\partial^2 u_L}{\partial P_A^2} < 0$ as in (20). Thus, we have

$$\frac{\hat{DP}_A}{\hat{D\alpha}} = \frac{\partial^2 u_A}{\partial P_A \partial \alpha} > 0.$$ 

**Proof of Proposition 3.9**

Suppose that the total number of lobbying groups that approach party A is equal to $N$ ($N \geq 2$). Party A will choose the policy $P_A$ in order to maximise the joint utility of all the lobby groups:

$$\max_{P_A} U = -s \sum_{k=1}^{N} (P_A - p_k)^2 - (1 - s) \sum_{k=1}^{N} p_k^2 - \sum_{k=1}^{N} c_k$$

This can be simplified as:

$$\max_{P_A} U = -NsP_A^2 + 2sP_A \sum_{k=1}^{N} p_k - \sum_{k=1}^{N} p_k^2 - \sum_{k=1}^{N} c_k$$

Noting that $\sum_{k=1}^{N} c_k = C_A$, we differentiate this utility function with respect to $P_A$ to obtain:

$$\frac{\partial U}{\partial P_A} = N \Omega_A P_A^3 - 2 \Omega_A P_A^2 \sum_{k=1}^{N} p_k - (Ns + \Phi_A) P_A + s \sum_{k=1}^{N} p_k = 0$$

Dividing both sides of this equation by $N$, we get:

$$\frac{\Omega_A P_A^3 - 2 \Omega_A P_A^2 \bar{p}}{N} - \left( \frac{Ns + \frac{\Phi_A}{N}}{N} \right) P_A + sp = 0$$

26
where \( \bar{p} = \frac{\sum_{k=1}^{N} p_k}{N} \). This equation is similar to equation (18) except that the party now takes the average policy standpoints of all groups into consideration. As a result, all the proof follows. In other words, there exists a collective equilibrium policy at which the political party maximises the joint utility of all the lobby groups.

Next, we consider two different interest groups, \( m \) and \( k \) with contribution levels \( c_m \) and \( c_k \) respectively. Denote \( P_A^* \) as the equilibrium policy conducted by the political party. In the first instance, suppose that the political party cares equally about the utility of each group, then:

\[
-s(p_m - P_A^*)^2 - (1 - s)p_m^2 - c_m = -s(p_k - P_A^*)^2 - (1 - s)p_k^2 - c_k
\]

This implies:

\[
c_k = s(p_m - P_A^*)^2 - s(p_k - P_A^*)^2 + (1 - s)(p_m^2 - p_k^2) + c_m
\]

Summing over \( k \) gives:

\[
\sum_{k=1}^{N} c_k = C_A = (N-1)s(p_m - P_A^*)^2 - s \sum_{k=1}^{N-1} (p_k - P_A^*)^2 + (1 - s) \sum_{k=1}^{N-1} (p_m^2 - p_k^2) + Nc_m
\]

Hence,

\[
c_m = \frac{C_A}{N} - \frac{(N - 1)s(p_m - P_A^*)^2 - s \sum_{k=1}^{N-1} (p_k - P_A^*)^2 + (1 - s) \sum_{k=1}^{N-1} (p_m^2 - p_k^2)}{N}
\]

This is the contribution made by a particular interest group \( m \). It can be seen that the first term on the right hand side is the average contribution. Depending on the sign of the second term, interest group \( m \) may have to contribute an amount that is greater or smaller than this average contribution.

Now suppose that the political party instead charges every lobby group an equal amount of contribution, i.e. \( c_m = c_k = \frac{C_A}{N} \). The difference in terms of utility between the two groups is:

\[
u_k - u_m = s \left[ (p_m - P_A^*)^2 - (p_k - P_A^*)^2 \right] + (1 - s)(p_m^2 - p_k^2)
\]

This can be simplified to:

\[
u_k - u_m = (p_m - p_k)(p_m + p_k - 2sP_A^*)
\]

Clearly, unless \( p_m = p_k \) or \( p_m + p_k = 2sP_A^* \), \( u_k - u_m \neq 0 \). In other words, the utility levels of the two groups are different.
Proof of Lemma 4.1

The expected utility maximisation problems for the lobby groups are:

\[
\max_{P_A} u_A = -s(P_A - q_A)^2 - (1 - s)(P_B - q_A)^2 - C_A
\]

(21)

and

\[
\max_{P_B} u_B = -s(P_A - q_B)^2 - (1 - s)(P_B - q_B)^2 - C_B
\]

(22)

where \(u_A\) and \(u_B\) denote the lobby groups’ utilities respectively. Hence, the first order necessary conditions for each case can be derived as:

\[
\frac{\partial u_A}{\partial P_A} = \Omega_A P_A^3 - 2\Omega_A q_A P_A^2 + (2q_A P_B - P_B^2)\Omega_A P_A - (s + \Phi_A)P_A + sq_A = 0
\]

(23)

and

\[
\frac{\partial u_B}{\partial P_B} = \Omega_B P_B^3 - 2\Omega_B q_B P_B^2 + (2q_B P_A - P_A^2)\Omega_B P_B - (1 - s + \Phi_B)P_B + (1 - s)q_B = 0
\]

(24)

Now we check for the second order conditions for having a maximum:

\[
\frac{\partial^2 u_A}{\partial P_A^2} = 5\Omega_A P_A^2 - 6q_A\Omega_A P_A + P_B(2q_A - P_B)\Omega_A - (s + \Phi_A) < 0
\]

(25)

and

\[
\frac{\partial^2 u_B}{\partial P_B^2} = 5\Omega_B P_B^2 - 6q_B\Omega_B P_B + P_A(2q_B - P_A)\Omega_B - (1 - s + \Phi_B) < 0
\]

(26)

It can be seen that these second order conditions are satisfied for a maximum given that \(1 \geq q_A \geq P_A \geq 0\) and \(-1 \leq q_B \leq P_B \leq 0\). In addition, they also indicate the existence and uniqueness of a solution \(P_A^*\) that solves (23) and \(P_B^*\) that solves (24) respectively. Indeed, for (23), with \(P_A \in [0,1]\) the range of value of the LHS is \([-\Phi_A - (P_B - 1)^2\Omega_A, sq_A]\). Because the LHS is decreasing in \(P_A\) and the range of value for the LHS includes both negative and positive region, (23) yields a unique solution in \([0,1]\) by single crossing property. Similar argument applies for (24).
Proof of Proposition 4.2

To that end, we first totally differentiate (21) and (22) and rearrange to get:

\[
\frac{\tilde{D} P_A}{D\theta_A} = \frac{\partial^2 u_A}{\partial P_A \partial \theta_A}
\]

\[
\frac{\tilde{D} P_B}{D\theta_B} = \frac{\partial^2 u_B}{\partial P_B \partial \theta_B}
\]

Note that we have \(\frac{\partial^2 u_A}{\partial P_A^2} < 0\) and \(\frac{\partial^2 u_B}{\partial P_B^2} < 0\) as per (25) and (26) respectively. From (23), partially differentiating with respect to \(\theta_A\) gives:

\[
\frac{\partial^2 u_A}{\partial P_A \partial \theta_A} = [P_A^3 - 2q_A P_A^2 + (2q_A P_B - P_B^2) P_A + 2P_A^2 - 2q_A P_A] \frac{\partial \Omega_A}{\partial \theta_A} - P_A \frac{\partial \Phi_A}{\partial \theta_A}
\]

Note that because \(P_A \leq q_A\), the term inside the square bracket is negative. In addition,

\[
\frac{\partial \Omega_A}{\partial \theta_A} = \frac{-R \alpha h (1 - \alpha) d}{(1 - \theta_A + R \theta_A \alpha h)^2} < 0
\]

and

\[
\frac{\partial \Phi_A}{\partial \theta_A} = \frac{R (1 - \alpha) d [1 - R \alpha h]}{(1 - \theta_A + R \theta_A \alpha h)^2} < 0
\]

where the second result holds as per Lemma 2.2. All these indicate \(\frac{\partial^2 u_A}{\partial P_A \partial \theta_A} > 0\). Thus, we have

\[
\frac{\tilde{D} P_A}{D\theta_A} = -\frac{\partial^2 u_A}{\partial P_A \partial \theta_A} > 0
\]

Similarly, partially differentiating (24) with respect to \(\theta_B\), we obtain:

\[
\frac{\partial^2 u_B}{\partial P_B \partial \theta_B} = [P_B^3 - 2q_B P_B^2 + (2q_B P_A - P_A^2) P_B + 2P_B^2 - 2q_B P_B] \frac{\partial \Omega_B}{\partial \theta_B} - P_B \frac{\partial \Phi_B}{\partial \theta_B}
\]

Note that because \(q_B \leq P_B \leq 0\), \(\frac{\partial \Omega_B}{\partial \theta_B} < 0\), and \(\frac{\partial \Phi_B}{\partial \theta_B} < 0\), we have \(\frac{\partial^2 u_B}{\partial P_B \partial \theta_B} < 0\). Thus, it is that

\[
\frac{\tilde{D} P_B}{D\theta_B} = -\frac{\partial^2 u_B}{\partial P_B \partial \theta_B} < 0
\]
To explore the impact of a change in $\theta_B$ on $P_A$, we partially differentiate (23) with respect to $\theta_B$ to get:

$$\frac{\partial^2 u_A}{\partial P_A \partial \theta_B} = 2\Omega_A P_A (q_A - P_B) \frac{\partial P_B}{\partial \theta_B} + (q_A - P_A) \frac{\partial s}{\partial \theta_B}$$

We have

$$\frac{\partial s}{\partial \theta_B} = \frac{\partial \Omega_B}{\partial \theta_B} P_B^2 + 2\Omega_B P_B \frac{\partial P_B}{\partial \theta_B}$$

Because $\frac{\partial \Omega_B}{\partial \theta_B} < 0$ and $\frac{\partial P_B}{\partial \theta_B} < 0$ as above then $\frac{\partial^2 u_A}{\partial P_A \partial \theta_B} < 0$. This means that $\frac{\partial P_A}{\partial \theta_B} < 0$. Similarly, we can show that $\frac{\partial P_B}{\partial \theta_A} > 0$. 

30